# Mortality projection incorporating model uncertainty

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## Extended Abstract

Researchers interested in longevity and mortality projection have available a wide variety of mortality projection models from which to choose (for example, Lee and Carter (1992), Booth, Maindonald and Smith (2002), Brouhns, Denuit, and Vermunt (2002), Currie, Durban, and Eilers (2004), De Jong and Tickle (2006), Renshaw and Haberman (2006), Cairns, Blake, and Dowd (2006), Delwarde, Denuit, and Eilers (2007), Hyndman and Ullah (2007)). Having been chosen, the favoured model is typically fitted against a suitable dataset and projected forward in time to produce not only expected future mortality rates but also, ideally, an estimate of the associated uncertainty in the form of a prediction interval, the width of which increases over time. However, different models not only yield different best estimates but also generate different prediction intervals, that is, the mortality projection uncertainty is model-dependent. In this paper, we describe a statistical approach to the quantification of mortality projection uncertainty that incorporates model uncertainty, that is, an approach that explicitly accounts for the fact that different projection models are available.

Bayesian statistical inference can provide a coherent fully probabilistic approach where inference and prediction statements are based on the posterior probability distribution of the unknown quantities given the observed data. Commonly these unknown quantities are parameters of a single statistical model, but the approach is sufficiently flexible to include uncertainty about the form of the model itself. Crucially, this allows predictive probability statements to be made which incorporate model uncertainty, and hence we consider such an approach to be well- suited to mortality projection under model uncertainty. For example, Abel et al (2010) illustrate how Bayesian methods can be used to provide more realistic projection uncertainty in an application to population forecasting using simple time series models.

Briefly, the Bayesian approach under model uncertainty updates a prior probability distribution over the models (in the form of probabilities or weights assigned to models) to a posterior distribution, in light of observed data. The posterior distribution accounts for how well the various models fit the observed historical data, and is used explicitly in weighting the models in projections. This approach is sometimes referred to as Bayesian model averaging – see Hoeting et al (1999) for details. The final projections fully integrate both model uncertainty and uncertainty about the parameters of the constituent models. Furthermore, as the approach is fully probabilistic, it is straightforward to also allow additional assumptions about future changes in key parameters and other aspects to be input into the projections, with the corresponding uncertainty being integrated in an entirely coherent fashion.

Although the principles of Bayesian inference (including under model uncertainty) are straightforward, practical methodology for incorporating probabilistic model uncertainty into mortality forecasts is currently underdeveloped. In this paper, we provide such a methodology. Initially, we focus on individual models, and develop Bayesian methodology for computing the predictive (forecast) distributions for various models. Some existing work on this area is described by Girosi and King (2008) and by Czado, Delwarde, and Denuit (2005) for particular models. The main contribution of our work is that we demonstrate how to effectively compute probabilistic projections *across* mortality projection models. We use a computational approach which involves separate Markov chain Monte Carlo (MCMC) generation of parameter values for each

model, together with a bridge sampling method (Meng and Wong, 1996) which uses the MCMC output to estimate the posterior model probabilities.

Our approach is illustrated on data from England and Wales. Using a diverse selection of different models, we present the projections provided by each model, and illustrate how the posterior model probabilities are computed, together with the resulting forecast arising from integrating over the models to account for model uncertainty. The integrated projection uncertainty provides a coherent and more realistic assessment of uncertainty than any corresponding analysis based upon a single model.

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