Stable Populations Revisited

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Extended abstract

The stable population model has first been defined by Alfred Lotka (1939). Since then it has been extensively investigated and extended, for example by Keyfitz and Preston (e.g. Keyfitz, 1968, 1985; Preston, 1982). The model describes the relation between age specific fertility and mortality rates on one hand and population growth and age structure on the other in a population where fertility and mortality have been fixed for a long period of time. Thus, the model shows which new age structure a population will adopt if the mortality or fertility rates in that population are changed and become frozen on a new level. According to the standard theory of demographic transition, a population in the initial phase, before mortality and fertility begin to decline, follows a stable population. Similarly, a population in the final phase of the demographic transition, when mortality and fertility are low, follows a stable population. During the actual transition period the stable population would not apply because changes take place and because the age structure lags behind.

We collected data on the age structure, mortality, and fertility of the Swedish population from Statistics Sweden and the Human Mortality Database to investigate some aspects of the Swedish population at 1900 and onwards. We constructed and analyzed demographic projection matrices for the age-structured population to calculate the stable age distribution. First we compared the actual age-structure in 1900 $c(a,1900)$ and compared it to the stable population determined by the fertility and mortality of 1900, $C(a,1900)$ and found a very good agreement between the actual population age structure and the corresponding stable population age structure for both sexes combined (Figure 1). The figure shows the proportion of each group on the total population in percent. Similarly, we compared in a second step the two structures of the population in 2000, $c(a,2000)$, and $C(a,2000)$. For this year, however, we found a clear divergence between actual and stable age structure, indicating that the actual age distribution is lagging behind and has not yet adopted the new stable shape that corresponds to the lower mortality and fertility that have been achieved during the previous century.

Secondly, starting again with the data at 1900, we investigated the relative impact of a decreased fertility and a decreased mortality on the future age distribution. We did that by first comparing the age distribution of the stable population at 1900, $C(a,1900)$, with the stable population that would result if the fertility rates remained unchanged but the mortality rates were fixed at the lower level of 2000, $C_{m2000}(a,1900)$ (Figure 2). Although the age distribution is changed by the declining mortality, the changes are not nearly as big as the actual changes. We then did the opposite, namely calculated the stable population that would result from the combination of the 1900 mortality rates and the 2000 fertility rates, $C_{f2000}(a,1900)$, (Figure 3) and compare it again with the stable population of 1900, $C(a,1900)$. This resulted in drastic changes in the age distribution and the result resembles the actual changes to the age distribution that have taken place during the 1900's. The results are summarized in Table 1. The conclusion from this exercise is that the big changes to the age distribution that have taken place during the previous century largely are the result of lowered fertility, not of increased life expectancy.

Table 1. Comparing structures of the population under different scenarios

1) Stable (1900) vs Observed (1900)	$C(a,1900) = c(a,1900)$	Structures of the populations, stable and observed in 1900, are equally looking of a young population
2) Stable (2000) vs Observed (2000)	$C(a,2000) \neq c(a,2000)$	Structures of the populations, stable and observed in 2000, are different, with the stable further advanced in the aging process
3) Stable fertility 1900 vs Stable (1900)	$C_{m2000}(a,1900) = C(a,1900)$	Changing only mortality doesn't change the structure of the population
4) Stable mortality 1900 vs Stable (1900)	$C_{f2000}(a,1900) \neq C(a,1900)$	Changing only fertility drastically changes the structure of the population

When looking at the mathematics behind the stable population, it is possible to see that the intrinsic growth rate plays a very important role in determining these changes. Further results and a discussion of the implications will be ready for the EPC meeting.

Figure 1: Population age structures of observed population of 1900 and Stable population of 1900

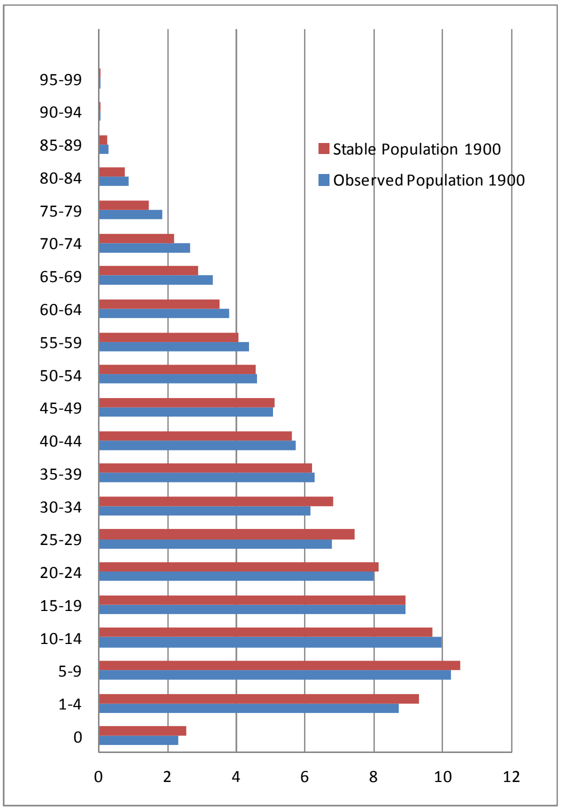


Figure 2: Population age structures of stable population of 1900 and the stable population with 2000 mortality and 1900 fertility

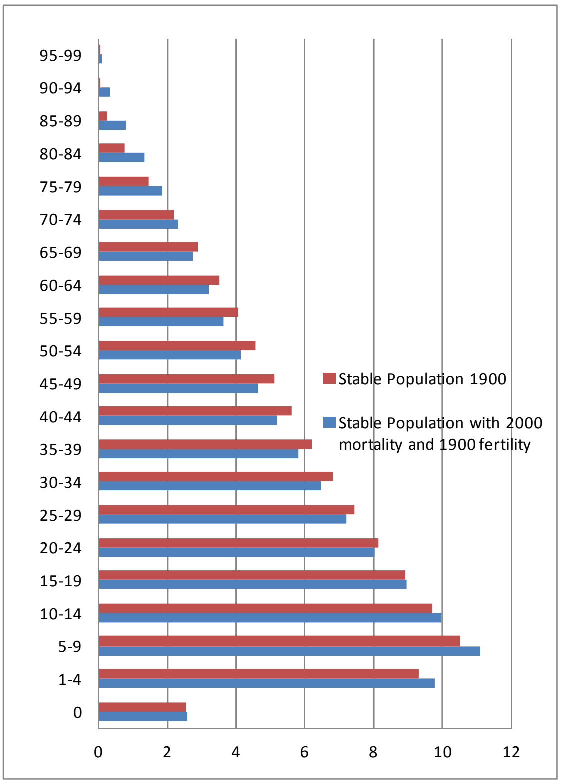
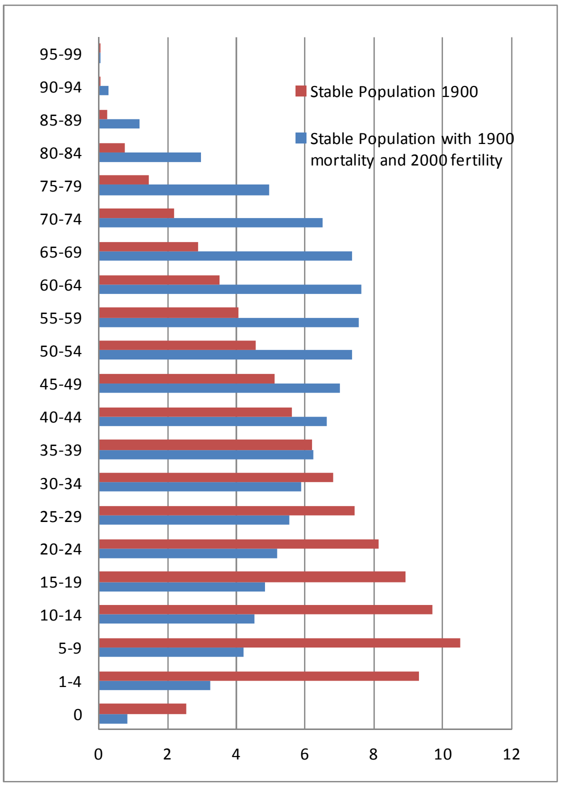


Figure 3: Population age structures of stable population of 1900 and the stable population with 1900 mortality and 2000 fertility



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