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On the Slope Information of Fertility Measures

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Outline of the Presentation

- Motivation of the study
- Basic ideas
- Cohort fertility predictors
- Source of estimation bias
- Slope information of fertility measures
- Conclusion / Remarks

- Can the Bongaarts-Feeney (1998) adjusted measure **help** to predict **cohort fertility**?

Pros of the BF:

- an **attractively simple** formula
- ASFRs are **available** for many countries

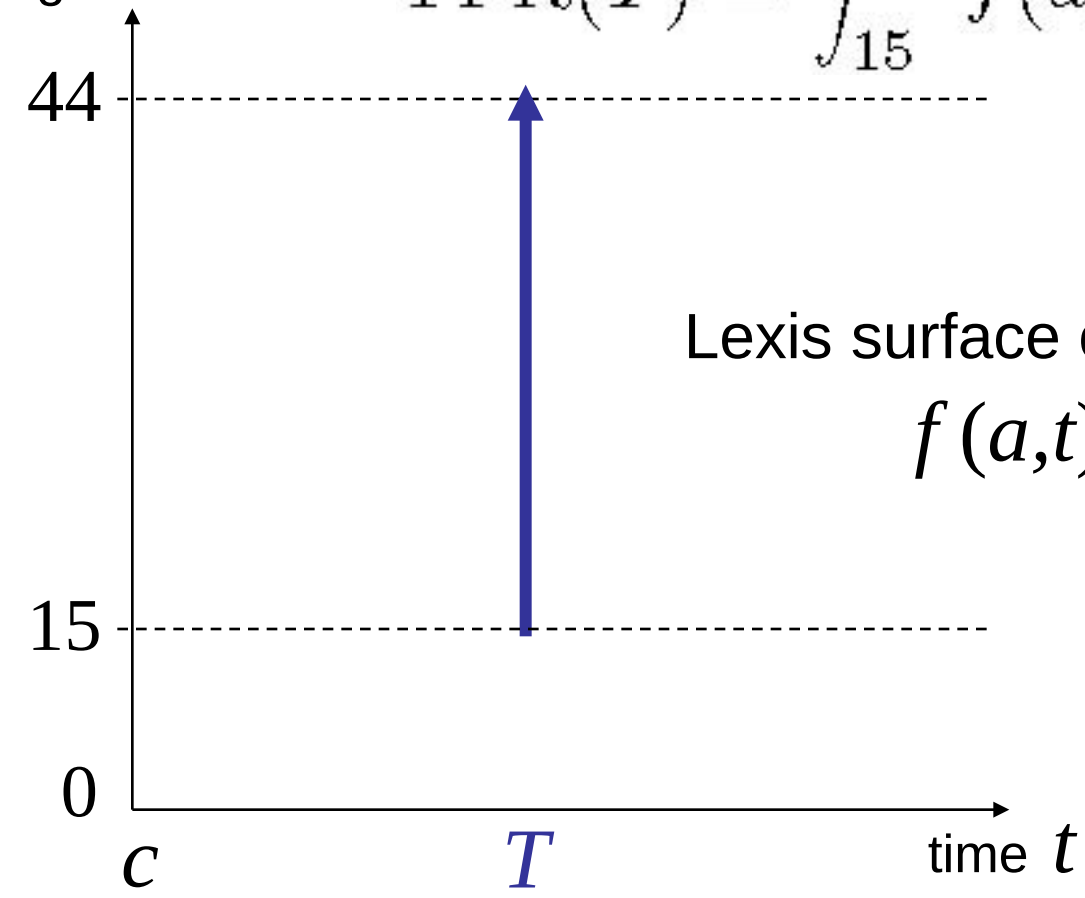
Cons of the BF:

- the **exclusion** of past fertility process
- the use of **incidence rates** for **non-repeatable** events
- the **unrealistic** assumption
- the **fluctuant** feature in its time series
- the obvious **discrepancy** from the time series of CFR

- Key: **how the question is perceived**.

the period-cohort relationship

$$\mathrm{TFR}(T) = \int_{15}^{44} f(a,T)\, da$$

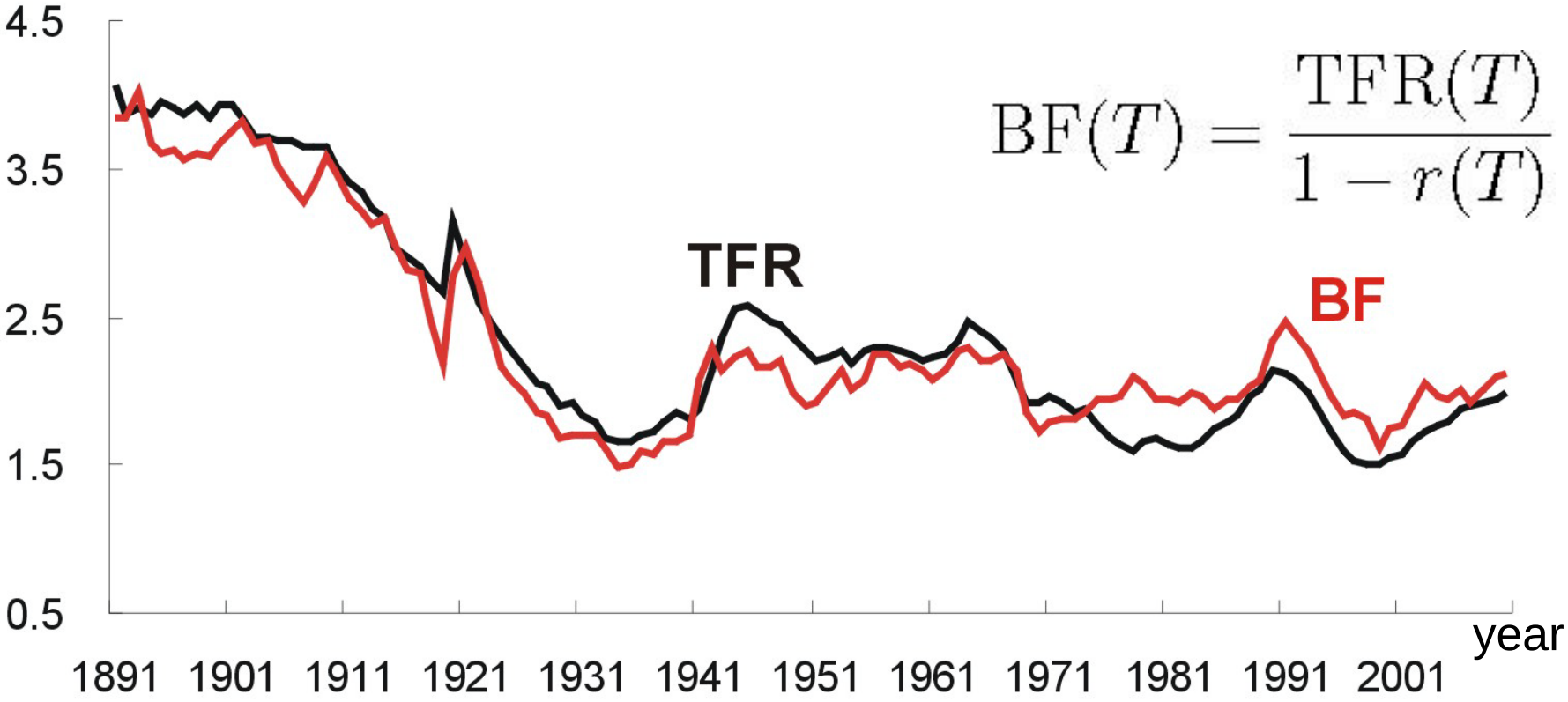


Lexis surface of ASFR:

$f(a,t)$

the period-cohort relationship

Sweden: 1891-2010 (all births combined)



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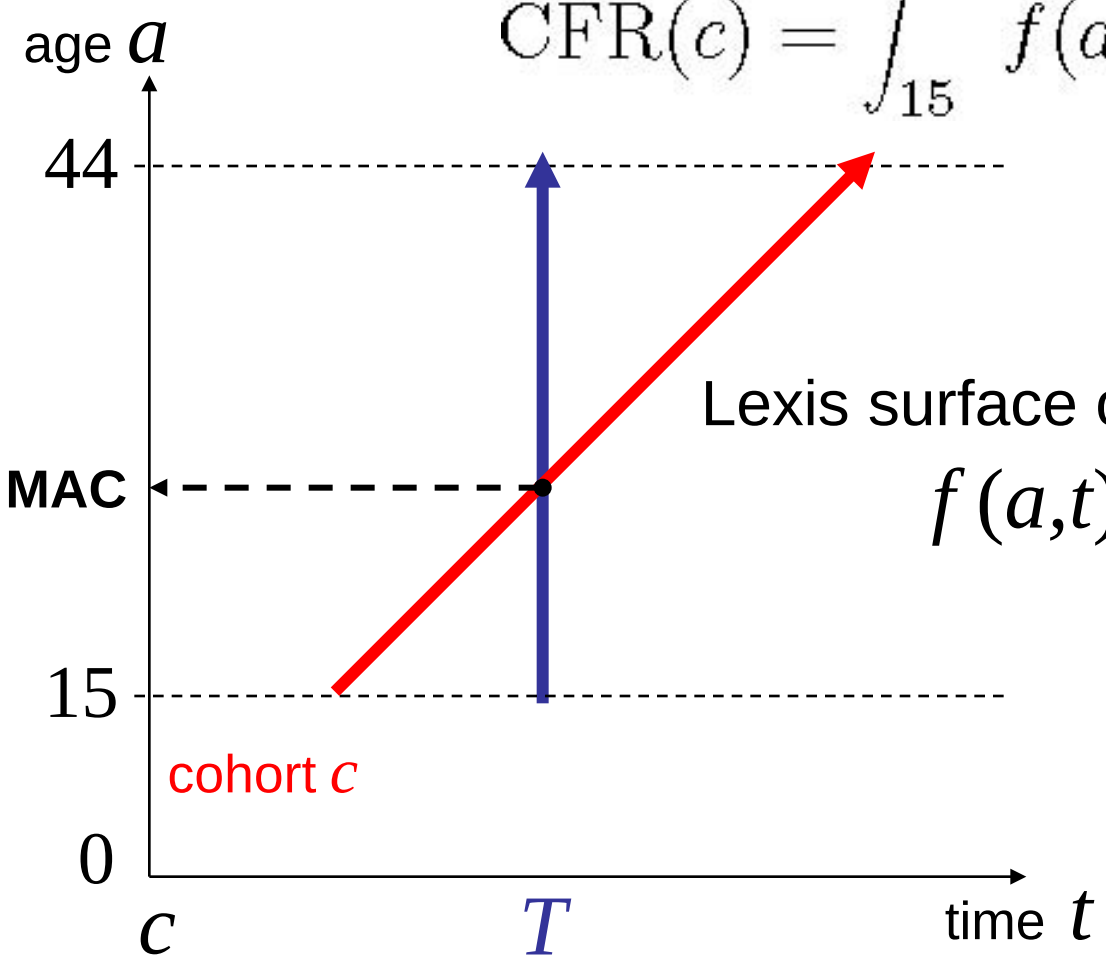
$$\mathrm{CFR}(c) = \int_{15}^{44} f(a, c+a)\ da$$



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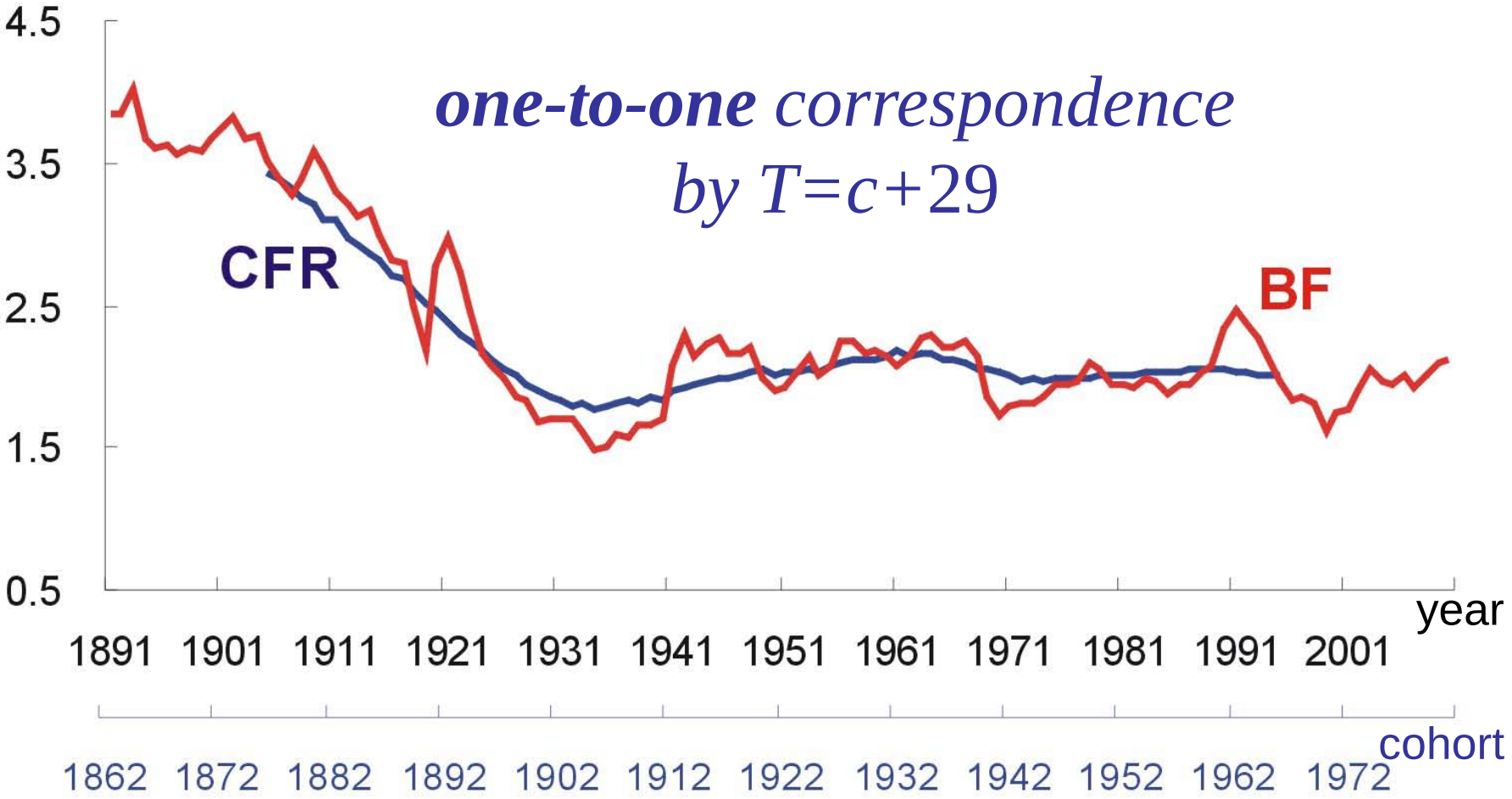

the period-cohort relationship

A Simple Math Identity

$$\mathrm{TFR}(T) = \int_{15}^{44} f(a,T)\ da$$

$$f(a,T)/\mathrm{TFR}(T) = p(a,T)$$

period fertility proportion

$$\mathrm{CFR}(c) = \int_{15}^{44} f(a,c+a)\ da$$

$$f(a,T) = \mathrm{TFR}(t)\ p(a,T)$$

$$= \int_{15}^{44} \mathrm{TFR}(c+a)\ p(a,c+a)\ da$$

$$= \int_{15}^{44} \frac{\mathrm{TFR}(c+a)}{1-r(c+a)} [1-r(c+a)]\ p(a,c+a)\ da$$

$$= \int_{15}^{44} \mathrm{BF}(c+a)\ w(a,c+a)\ da$$

a linear combination of BFs

$$\mathrm{CFR}(c) = \int_{15}^{44} \mathrm{BF}(c+a)\; w(a, c+a)\; da$$

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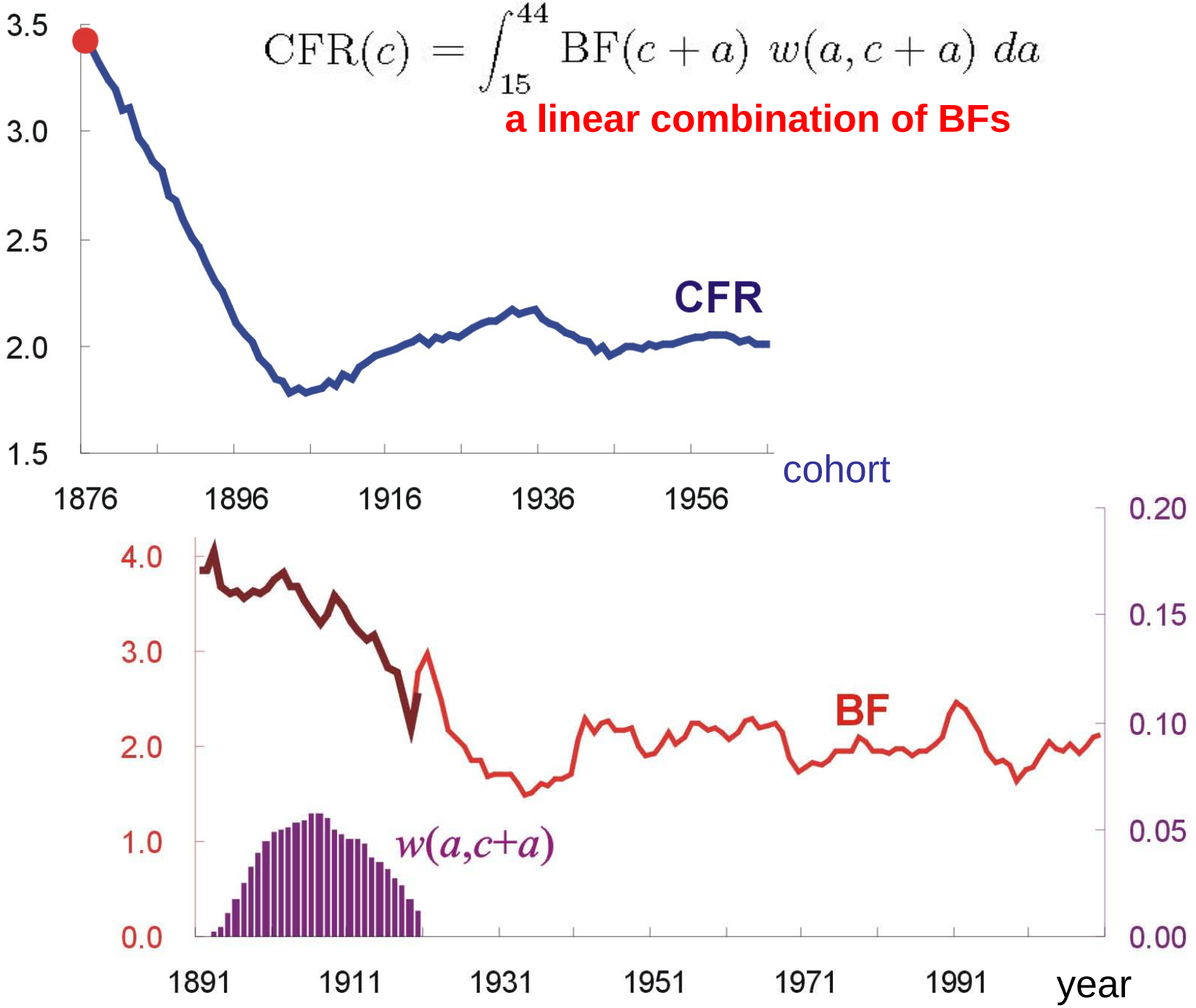

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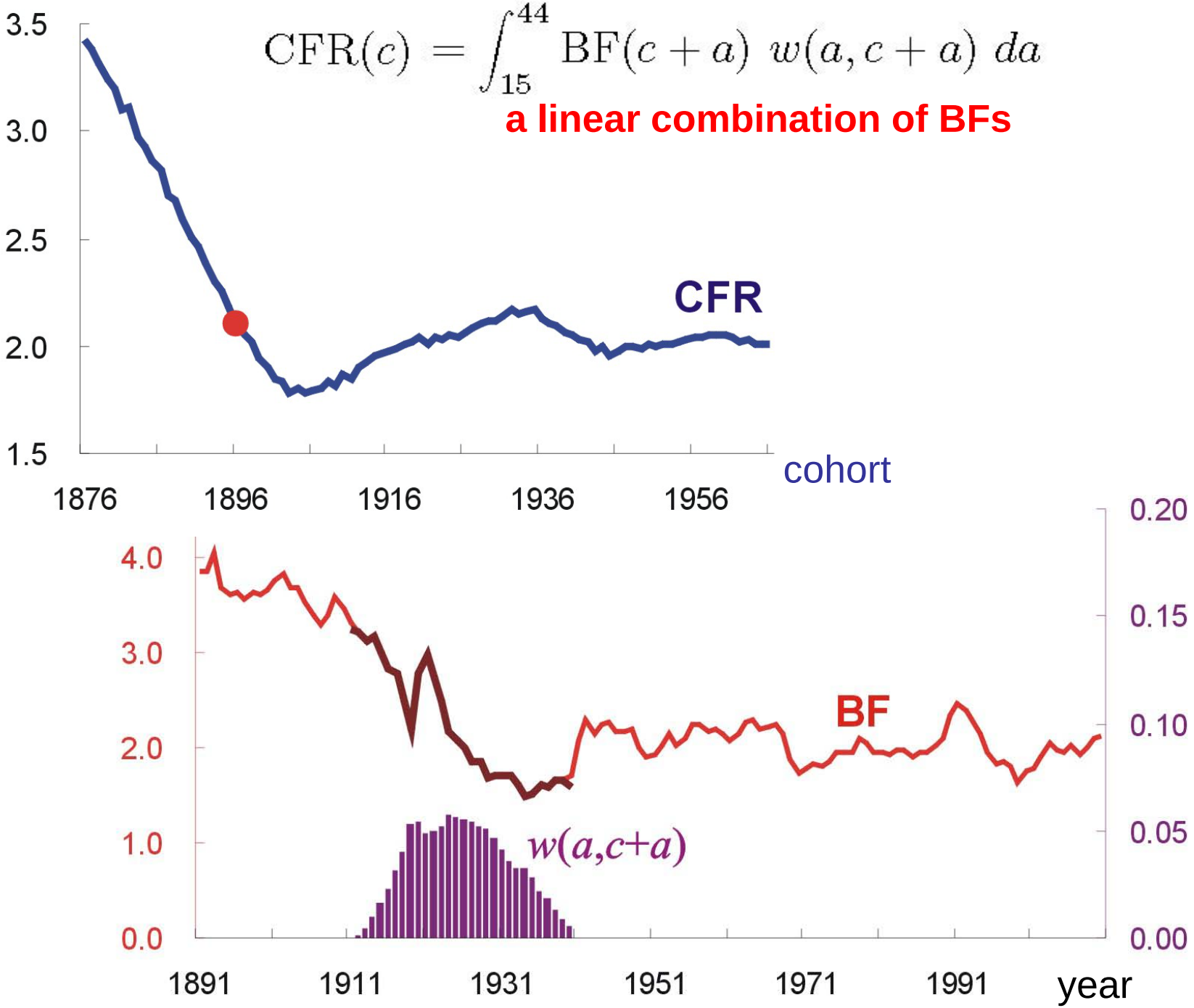

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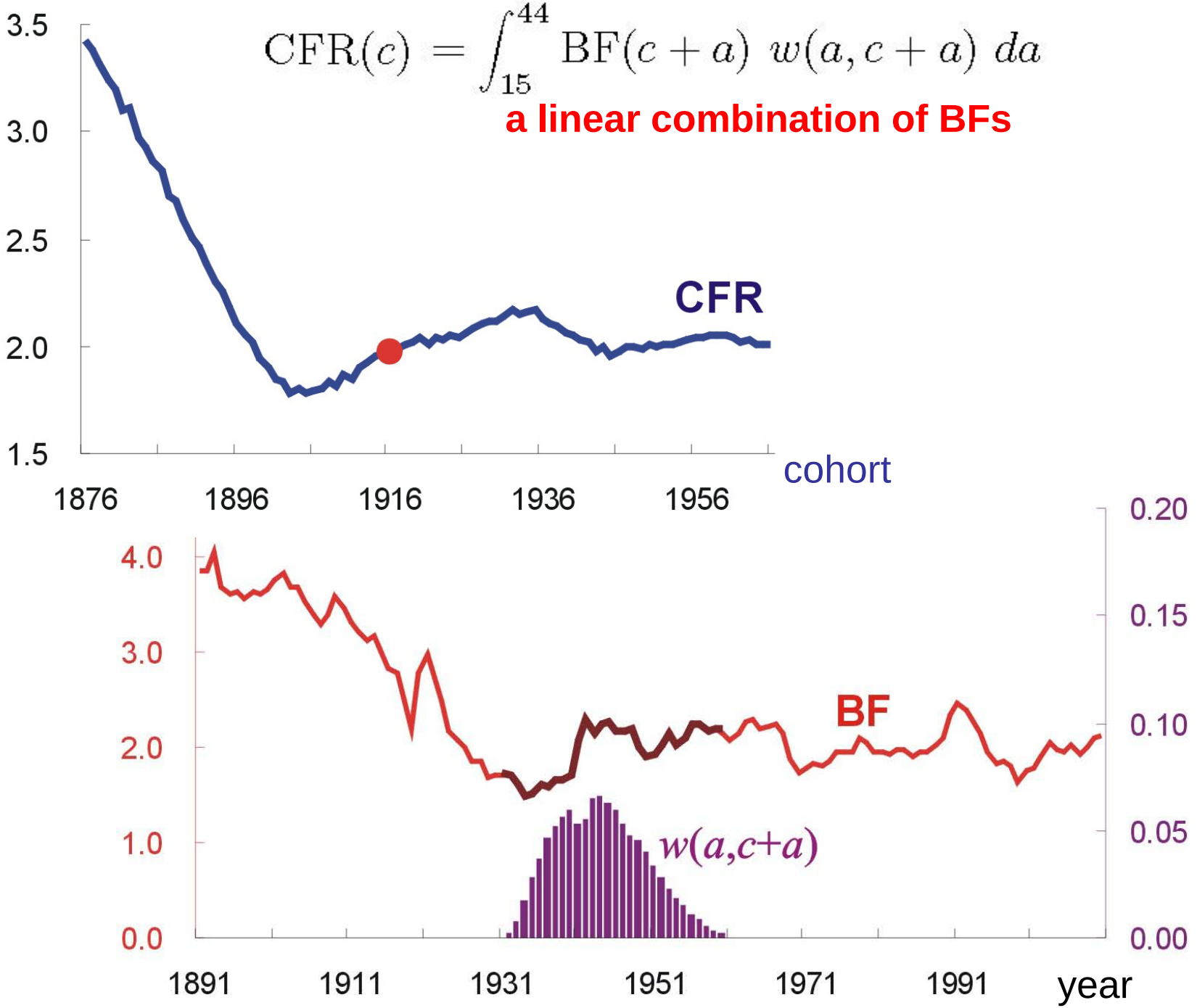

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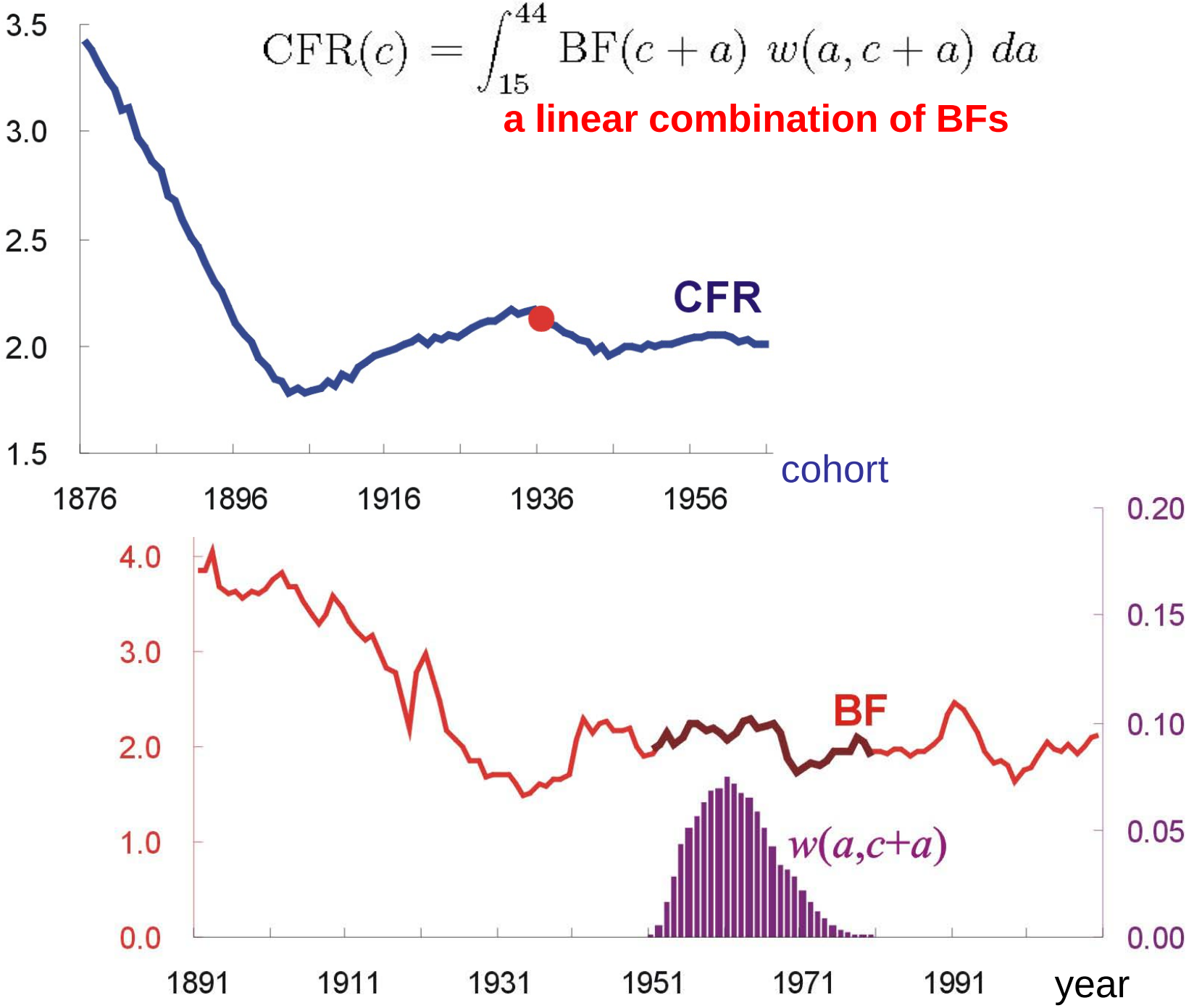


$$\mathrm{CFR}(c) = \int_{15}^{44} \mathrm{BF}(c+a)\ w(a, c+a)\ da$$

first shown in Bongaarts and Feeney (2006), but not further utilized in developing a CFR predictor.

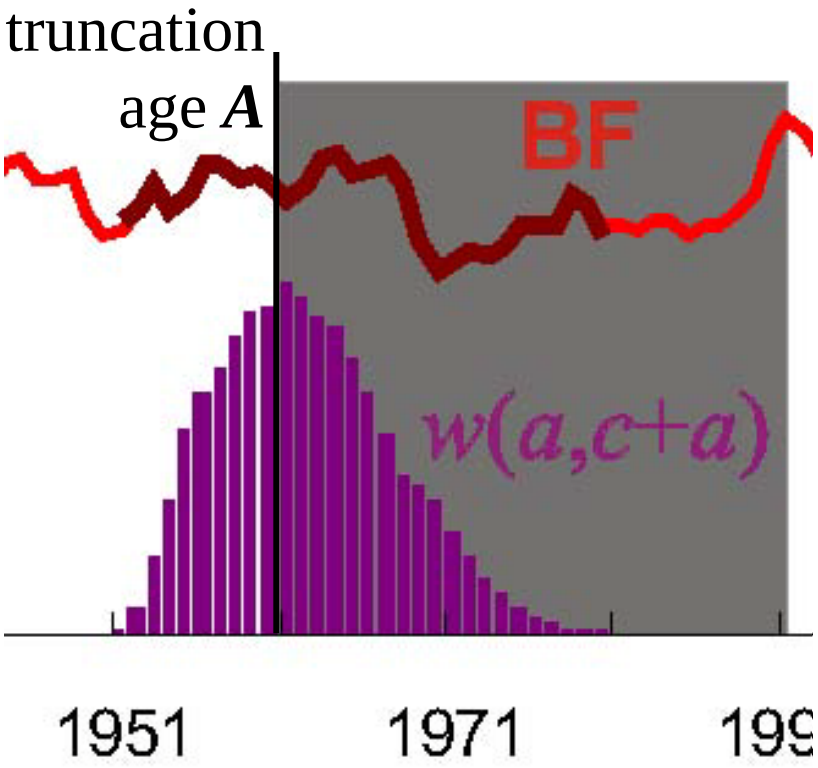
- The **many-to-one** perspective can **waive most** (if not all) **previous challenges**:
  - ✓ past fertility process of the target cohort is now **incorporated**;
  - ✓ the aim is to construct a **real** (rather than **synthetic**) cohort measure;
  - ✓ applicable to **any birth order** and equivalent to **smoothing** the time series of BF;
  - ✓ **empirical evidence** shows its usefulness.

Cohort fertility predictors

- When data of cohort childbearing is **truncated** at age A,

$$\mathrm{CFR}(c) = \int_{15}^{A} \mathrm{BF}(c+a)\; w(a, c+a)\; da + \int_{A}^{44} \mathrm{BF}(c+a)\; w(a, c+a)\; da$$

observed **unfinished**



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observed **unfinished**

assumed constant at a value of

the average of observed BFs

or

the latest BF

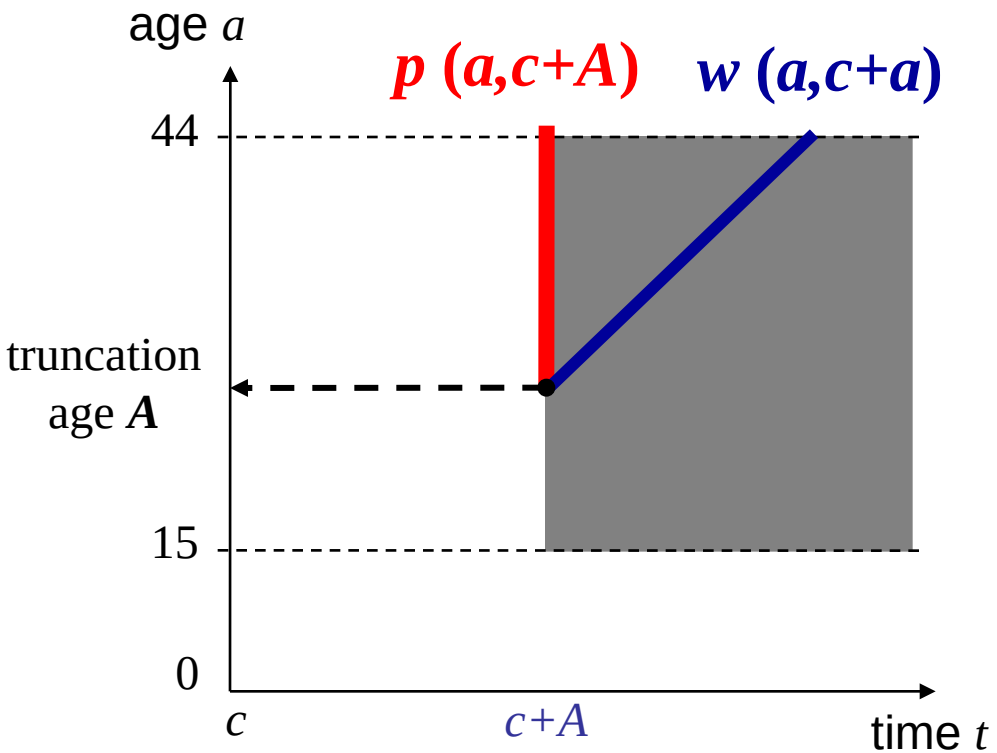


Table 2: Data from the Human Fertility Database and the Eurostat

country	all-birth-combined		parity-specific	
	periods	completed cohorts	periods	completed cohorts
from the Human Fertility Database				
Austria	1951–2010	1936–1966 (31)	1984–2010	NA
Bulgaria	1947–2009	1932–1965 (34)	1947–2009	1932–1965 (34)
Canada	1921–2007	1906–1963 (58)	1944–2007	1929–1963 (35)
Czech Republic	1950–2009	1935–1965 (31)	1950–2009	1935–1965 (31)
Estonia	1959–2009	1944–1965 (22)	1959–2009	1944–1965 (22)
Finland	1939–2009	1924–1965 (42)	1982–2009	NA
France	1946–2009	1931–1965 (35)	NA	NA
Germany				
East	1956–2010	1941–1966 (26)	1956–1989	NI
West	1956–2010	1941–1966 (26)	NA	NA
Hungary	1950–2009	1935–1965 (31)	1952–2009	1937–1965 (29)
Lithuania	1959–2009	1944–1965 (22)	1970–2009	1955–1965 (11)
Netherlands	1950–2009	1935–1965 (31)	1950–2009	1935–1965 (31)
Portugal	1940–2009	1925–1965 (41)	1959–2009	1944–1965 (22)
Russia	1959–2009	1944–1965 (22)	1959–2009	1944–1965 (22)
Slovakia	1950–2009	1935–1965 (31)	1950–2009	1935–1965 (31)
Sweden	1891–2010	1876–1966 (91)	1970–2010	1955–1966 (12)
Switzerland	1932–2009	1917–1965 (49)	1998–2009	NA
U.K.				
England/Wales	1938–2009	1923–1965 (43)	NA	NA
Scotland	1945–2009	1930–1965 (36)	NA	NA
U.S.	1933–2007	1918–1963 (46)	1933–2007	1918–1963 (46)
from the Eurostat				
Belgium*	1954–2009	1939–1965 (27)	NA	NA
Denmark	1950–2010	1935–1966 (32)	NA	NA
Greece	1961–2010	1946–1966 (21)	NA	NA
Iceland	1963–2010	1948–1966 (19)	NA	NA
Italy	1952–2008	1937–1964 (28)	NA	NA
Norway	1961–2010	1946–1966 (21)	NA	NA
Spain	1971–2010	1956–1966 (11)	NA	NA

Note: 1. When a country is included in both databases, we prioritize data from the Human Fertility Database.

- **905 and 331 completed cohorts for non-parity and parity specific data from 27 countries/areas, including Canada, the U.S., and 23 European countries.**

- **For each completed cohorts, a number of experiments can be implemented by varying the truncation age A which corresponds to a particular completed proportions.**

Empirical evaluation

- Adopt the **prediction error** index as:

$$\mathbf{PE} = \frac{\text{est. CFR} - \textbf{true CFR}}{\textbf{true CFR} - \text{obs. CFR}} * 100\%$$

  - ✓ how much of the **unfinished fertility** has not been correctly estimated

- ✓ For example: est. CFR=1.8 **true CFR**=2.0
  - obs. CFR=0.8 completed proportion =40% PE=−16.67%
  - obs. CFR=1.2 completed proportion =60% PE=−25.00%

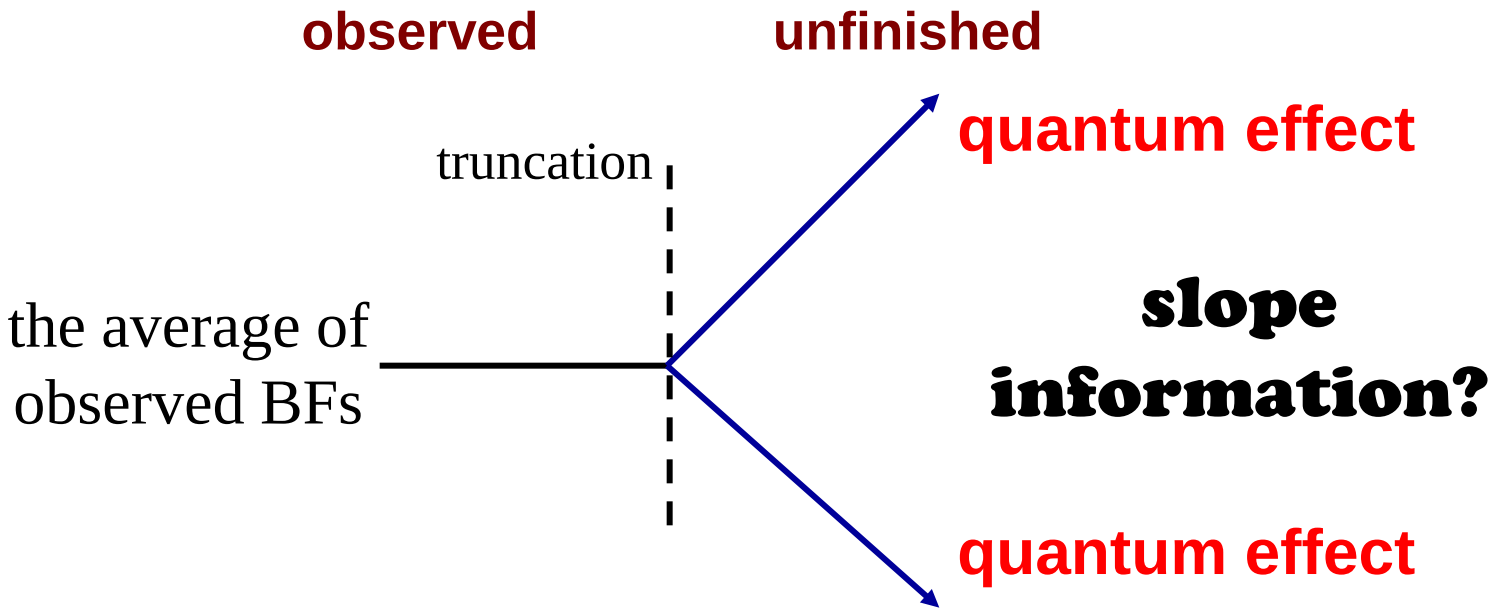
- Take **20%** as the PE cut point

Mean absolute PE by method, completed proportion, and birth order

birth order	N	**Proportion Inflation**	Freeze Rate	Linear Extrapolation	TFR	BF
		completed proportion in **[10, 30)**				
all	2,759	12.74	17.03	18.27		
1	677	5.05	9.47	12.04		
2	833	7.43	10.92	15.26		
3+	998	34.23	31.29	28.06		
		completed proportion in **[30, 50)**				
all	2,440	13.38	16.96	16.42		
1	597	5.28	10.76	10.90		
2	687	8.02	12.52	15.17		
3+	843	32.56	26.17	23.81		
		completed proportion in **[50, 65)**			***mean age***	
all	1,999	14.69	17.15	15.91	29.32	19.58
1	499	6.10	11.87	10.92	23.63	14.08
2	576	9.30	13.77	14.87	28.98	17.51
3+	673	34.12	23.42	22.07	34.93	34.48

- **P.I.** outperforms conventional methods except on order 3+.
- **L.E.** is the best on order 3+, but its performance is classified "poor" based on the **20%** standard.

- Cheng and Goldstein (2011) did some further investigation on these results and found:
  - ✓ tempo effect is similar across birth orders;
  - ✓ **quantum effect** is prominent for higher orders;
  - ✓ our **tempo-adjusted** predictor fails to deal with the quantum effect well, but neither do others.



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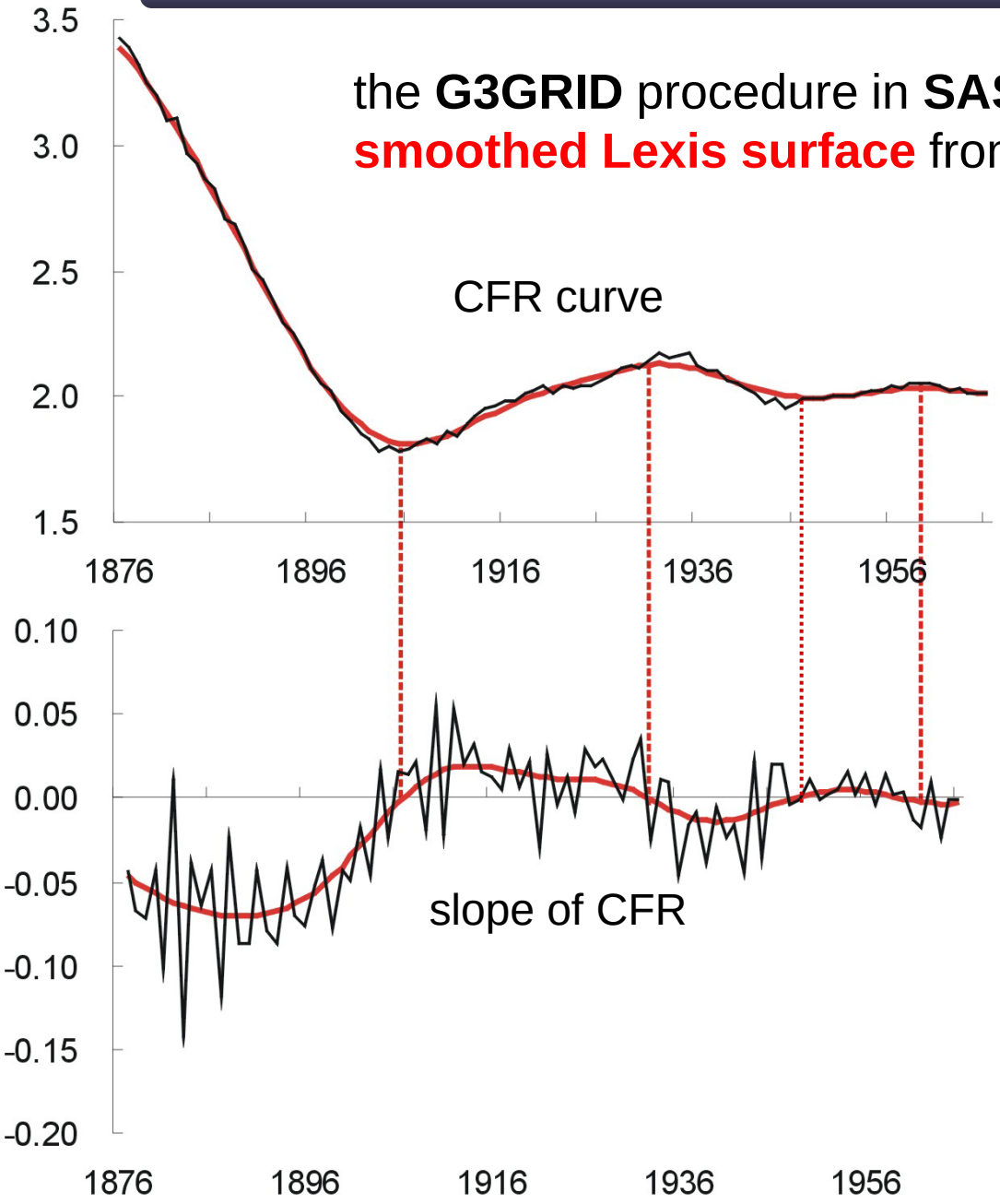

the **G3GRID** procedure in **SAS code** to create a **smoothed Lexis surface** from the original data

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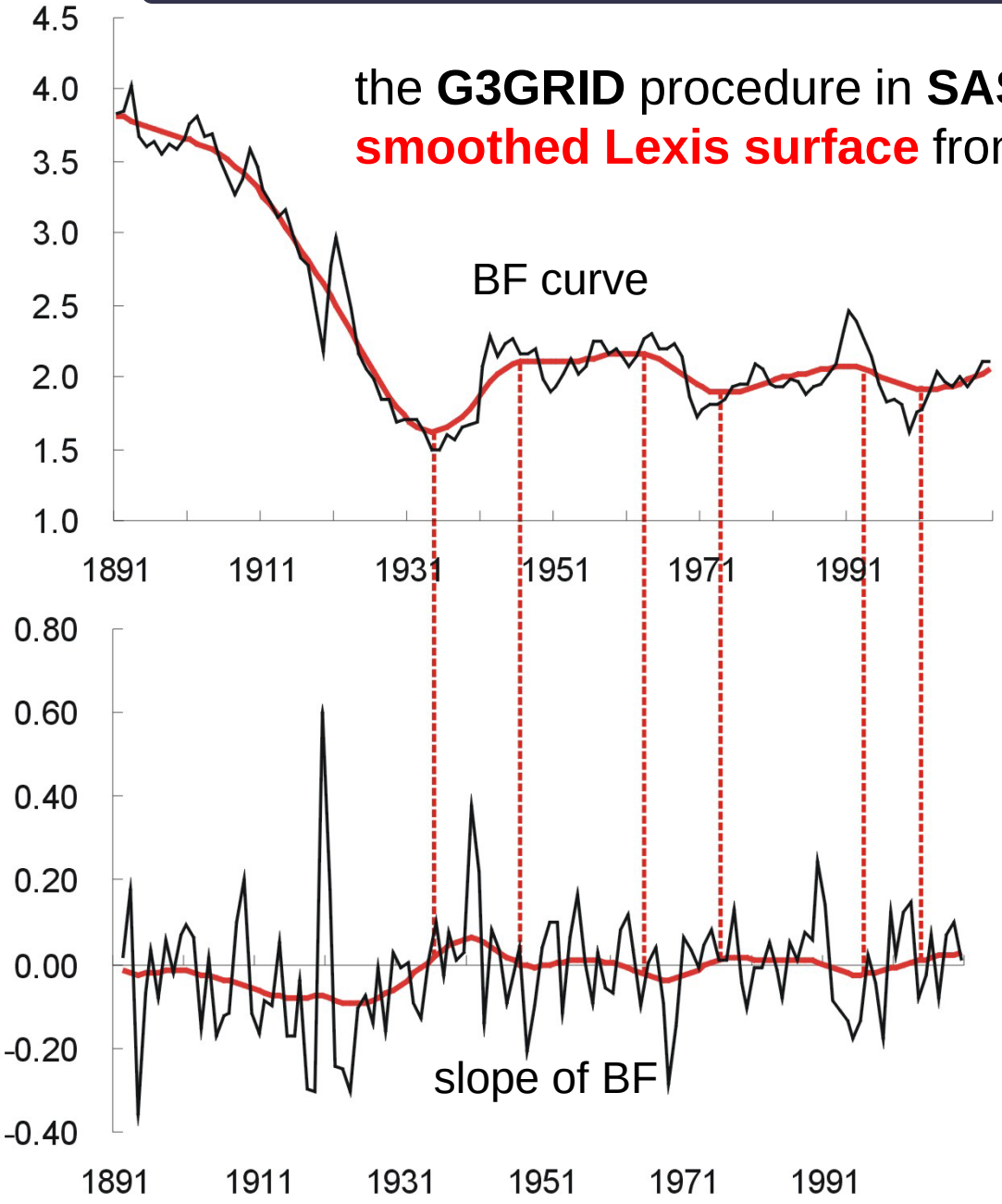

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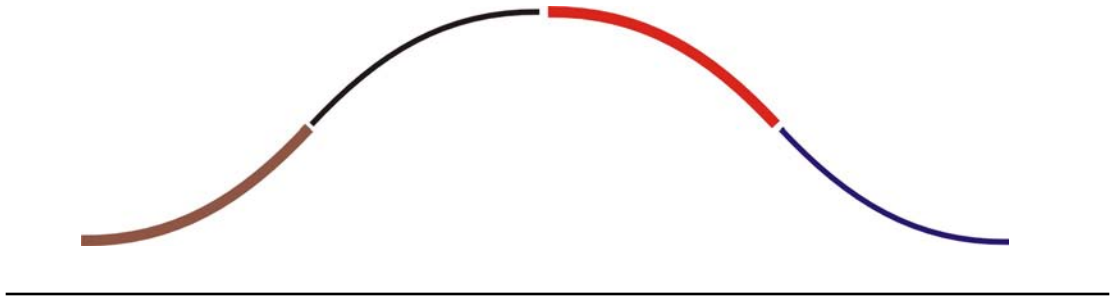
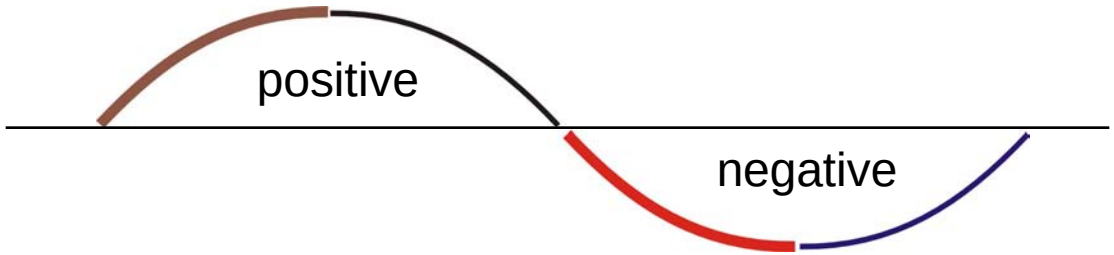
BF Curve



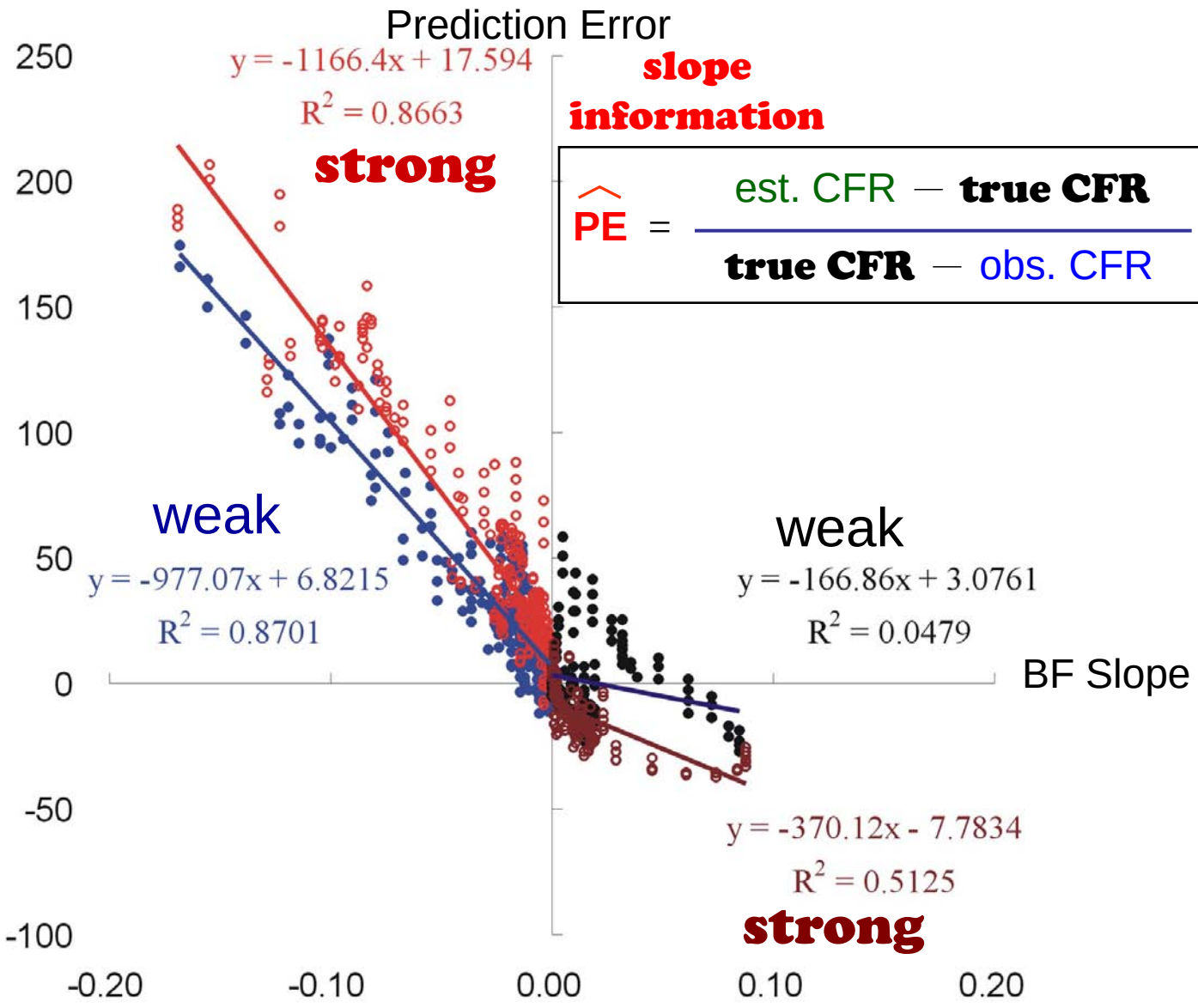
Slope of BF



Quantum effect

strong weak **strong** weak

Birth order=3+, completed proportion=[30,50)



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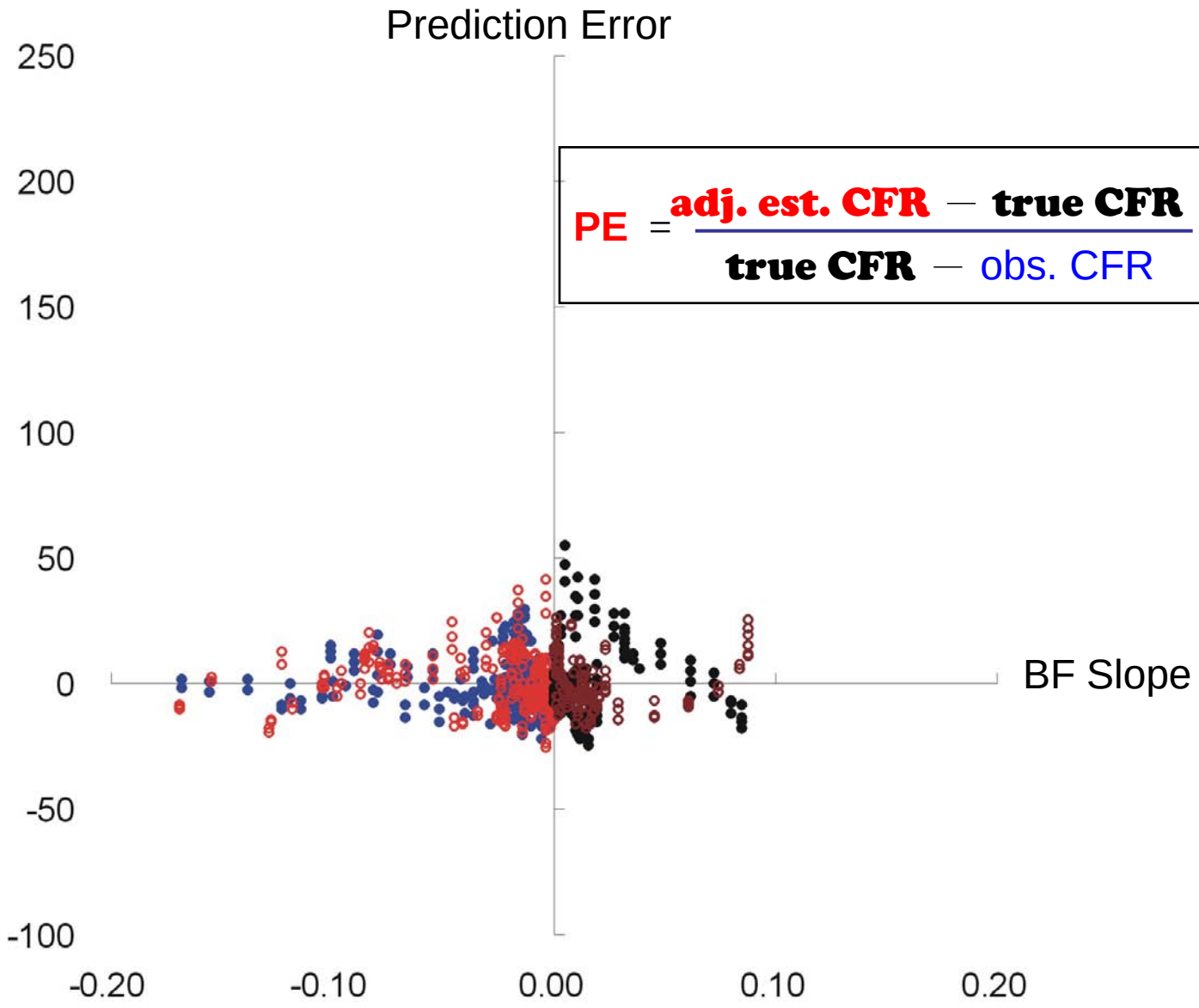

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- **adj. P.I.** outperforms conventional methods regardless of birth order

Conclusion / Remarks

- The period-cohort correspondence is better perceived as a **many-to-one** relationship.
- The **BF** measure does help to predict **CFR**.
- **All** CFR estimators we investigated suffer from the **quantum** effect.
- The **BF slope** information does help to eliminate the bias caused by the **quantum** effect.
- The **auxiliary regressions** need a justification (maybe some simulations?).
- The **BF slope** may be **biased** for periods near the data end (need some further research on this).