Measuring the Morphology of the Life table

Jon Anson
Dept. Of Social Work, Ben-Gurion University of the Negev
84105 Beer Sheva, Israel

As living standards improve, mortality declines, and the age-pattern of human mortality changes. Yet, while much of human mortality decline can be related, directly and indirectly, to changes in the material standards of living, differences in human mortality, and in the shape of the mortality curve, cannot be explained in terms of material standards of living alone. Different social conditions, levels of technology and patterns of social relations, between social groups and between men and women, will result in a different age structuring of mortality, even in populations with a similar average level of mortality. How, then, can we describe and explain these differences in the shape of the mortality curve?

There is a pattern of age specific mortality which, under normal circumstances, is common to all life tables: the rapid decline in infancy and childhood, the minimum attained at about age 10, the increase to a possible plateau in early adulthood, and the gradual rise in the level of mortality as senescence is approached. As mortality declines there is a growing concentration of deaths at the upper end of the age scale, with a consequent pushing back of the modal age at death (Fries, 1983; Wilmoth & Horiuchi, 1999). However, not all populations follow the same path. In previous work (Anson, 1991, 1992) we have suggested that human life tables can be located in a two-dimensional space, and that just two parameters are required in order to identify a life table uniquely and to distinguish it from all others: one parameter to describe the general level of mortality in the population, and one to describe the specific shape (morphology) of the curve, net of the level of mortality. The present paper considers how this second dimension may be measured and how it may be related to the specific conditions in which the population lives.

A parametric reproduction of the mortality curve requires between five (Siler, 1983) to eight parameters (Heligman and Pollard, 1980; Mode & Busby, 1988), but many of these are redundant given the uniform underlying form of the curve. In practice, just two parameters are sufficient in order to identify the mortality curve, with a third to distinguish between the male and female curves of a given population (Lederman & Breas, 1959; Brass, 1971; Anson, 1991, 1993). There have, indeed, been various attempts to define a measure of the shape of the mortality curve. Some of these encompass the whole of the life table, such as the

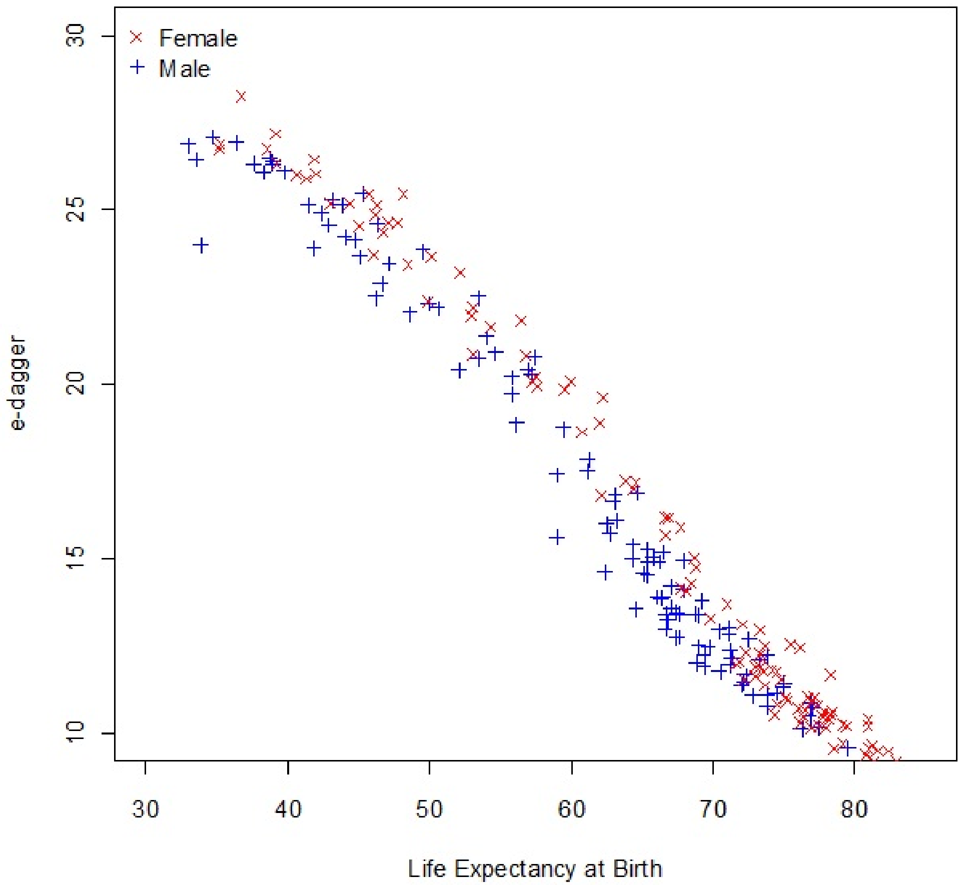
Keyfitz-Golini measure of life-table elasticity, or rectangularity, H (Leser, 1955; Keyfitz, 1977); life disparity, $e^{\dagger}$ (Vaupel & Canudas-Romo, 2002; Shkolnikov et al., 2011); the entropy of the age distribution of the life table (Demetrius, 1979) or of the mortality distribution (Anson, 2002), and measures of life table inequality (Silber, 1992, Shkolnikov, 2003) . Others are more localised, such as the rate of mortality increase in later life (Thatcher et al. 2010, Tuljapurkar & Edwards, 2011).

Almost without exception, however, all these measures are so closely correlated with the level of mortality as to provide little more than a description of the change in the form of the curve as mortality declines (for a review, see Anson, 2002). Figure 1 plots two such measures, $e^{\dagger}$ ($\int l_x \cdot \ln(l_x)dx$) and H_d, the entropy of the d_x curve ($\sum_{dx} \cdot \ln(dx)$) against life expectancy for a random sample of 109 male and female life tables taken from the Human Mortality Database (www.mortality.org, data downloaded 11/10/2011). As life expectancy increases, $e^{\dagger}$ decreases consistently, with female tables having a marginally higher value of $e^{\dagger}$ than male tables, but otherwise the variation at any given level of e_0 appears to be minimal. The specific level of $e^{\dagger}$ would thus appear to be almost entirely determined by the average level of mortality in the population. H_d, by contrast, which measures the extent to which life-table deaths are evenly distributed over the age range, rises as life expectancy increases, up to a life expectancy of about 55, after which it begins to decline. There appears to be no difference between male and female tables in the level of H_d for any given e_0, but on the other hand there does seem to be considerable variation in the level of H_d between populations, particularly in the vicinity of the turning point. This implies differences in the way the mortality pattern changes as life expectancy increases, but this can only be seen when H_d is evaluated *net* of the specific value of e_0.

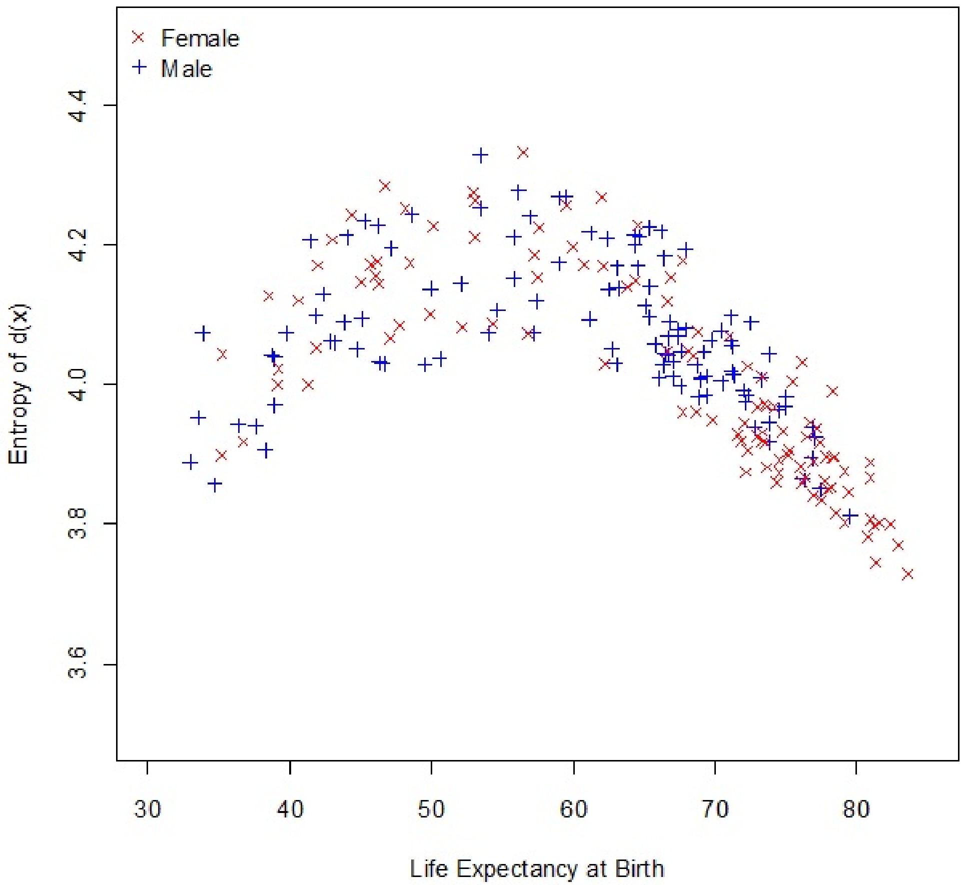
Thus, if we wish to locate life tables in a two-dimensional mortality space, we need to define the shape of the curve in a manner which is uncorrelated with the level of mortality. Otherwise, we are merely comparing mortality levels by another name. By contrast, distinguishing the dimensions of the mortality curve will contribute in a number of ways to the better understanding of a population's vital processes. First, by identifying the normal range of these dimensions, we will be better able to identify life tables of dubious reliability. Second, by identifying the relation between critical social parameters and the dimensions of the mortality curve, we will be in a better position to understand the dynamics of mortality in different populations, and in particular the conditions which give rise to an unduly high level of premature and avoidable mortality.

Figure 1: Morpholgy measures and the level of life expectancy at birth

a: $e^{\dagger}$ (e-dagger) by life expectancy



b: H_d (entropy of mortality distribution) by e_0



As mortality declines to ever lower levels, the difference between life tables is going to be more and more in the details of the distribution of mortality, and measures of these details are going to play an ever more important role. In the present paper we consider how these morphological measures may be used to compare the internal distribution of mortality within the life table, while yet allowing for the changes which necessarily occur as mortality declines. This will enable us to identify differences in the shapes of mortality and survivorship curves, that is, in the distribution of deaths over the life span, which are *not* a necessary corollary of the differences in the overall level of mortality. Such differences may then be related to differences in the conditions (social, economic, physical) in which the population lives its life, as will be illustrated using examples from various contemporary and historical populations, at the national and sub-national level.

Bibliography

Anson, J. 1991. Model mortality patterns: A parametric evaluation. *Population Studies*, **45**:137-153.

Anson, J. 1992. The second dimension: A proposed measure of the rectangularity of mortality curves. *Genus*, **48**:1-17.

Anson, J. 1993. Regional mortality differences in Britain, 1931-1987: A two-dimensional analysis. *Journal of Biosocial Science*, **25**:383-395.

Anson, J., 2002, "Of entropies and inequalities: Summary measures of the age distribution of mortality," in G. Wunsch, M. Mouchart & J. Duchêne (eds), *The Life Table: Modelling survival and death,"* Dordrecht: Kluwer.

Brass, W. 1971. On the scale of mortality, in W. Brass, *Biological Aspects of Demography*, London: Taylor & Francis

Demetrius, L. 1979, "Relations between demographic parameters," *Demography*, **16,** 329-338**.**

Fries, J. F. 1983. The Compression of Morbidity. *The Milbank Memorial Fund Quarterly.* **61**(3): 397-419.

Heligman, L. & Pollard, J. H. 1980. The age pattern of mortality. *Journal of the Institute of Actuaries,* **107**: 49-80

Keyfitz, N. 1977, *Introduction to the Mathematics of Population, with revisions,* Reading, MA, Addison-Wesley.

Lederman S. & Breas, J. 1959. Les dimensions de la mortalité. *Population*, **14**: 637-682.

Leser, C. E. V. 1955. “Variations in Mortality and Life Expectation,” *Population Studies*, **9(**1): 67-71.

Mode C. J. & Busby, R. C. 1988. An eight-parameter model of human mortality: The single decrement case. *Bulletin of Mathematical Biology*, **44**: 647-659.

Shkolnikov, V. M., Andreev, E. M., & Begun, A. Z. 2003. “Gini coefficient as a life table function. Computation from discrete data, decomposition of differences and empirical examples.” *Demographic Research*, **8**, article 11:305–58.

Shkolnikov, Vladimir M., Evgeny M. Andreev, Zhen Zhang, James Oeppen, and James W. Vaupel. 2011. "Losses of Expected Lifetime in the United States and Other Developed Countries: Methods and Empirical Analyses." *Demography* 48:211–239.

Silber, J. 1992, “Inequality in mortality: Measuring the contribution of various causes of death,” *Genus*, **48**:(3-4): 93-107.

Siler, W. 1983. Parameters of mortality in human populations with widely varying life spans. *Statistics in Medicine*, **2**: 373-380.

Thatcher, A. Roger, Siu Lan K. Cheung, Shiro Horiuchi, and Jean-Marie Robine. 2010. "The compression of deaths above the mode." *Demographic Research* 22:505-538.

Tuljapurkar, Shripad and Ryan D. Edwards. 2011. "Variance in death and its implications for modeling and forecasting mortality." *Demographic Research* 24:497-526.

Vaupel, JamesW. and Vladimir Canudas-Romo. 2002. Decomposing change in life expectancy: A bouquet of formulas in honor of Nathan Keyfitz’s 90th birthday,” *Demography*, **40**(2): 201-216.

Wilmoth, J. R. & Horiuchi, S. 1999. Rectangularization Revisited: Variability of Age at Death within Human Populations. *Demography*, **36**(4): 475-495.