

## **Sensitivity of future life expectancy at birth and at age 65 due to different mortality forecasting methods**

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### **Introduction**

With the rapid aging of the population, the importance of mortality forecasts has increased. That is, through the recent reforms to the pension systems in Europe – necessary for pension to remain affordable – the link between pensions and changes in life expectancy has never been that evident. Not only the monthly pension will depend on the remaining life expectancy when people retire, but in some countries benefit levels are linked to life expectancy, the pension age will be increased with an increasing life expectancy, or the years of contributions will be extended necessary for a full benefit as people live longer (OECD, 2007). An accurate modeling and projection of mortality rates and life expectancy is therefore of ever growing interest.

Current approaches to project future mortality differ by country but also within countries. An important issue is as to whether merely ‘objective’ extrapolation methods should be employed, or whether additional epidemiological information, through explanatory models and/or by including trends in other countries are preferable, despite additional subjectivity.

Despite the many studies on forecasting mortality, the comparison of different methods is less comprehensive. Recent studies do however extensively compare variants and extensions of one particular method, e.g. Booth et al. (2002, 2005, 2006), Shang et al. (2011) and Wang and Liu (2010). These recent studies show that a comparison of different variants and extensions of one approach does not automatically lead to the choice of one best variant. Moreover, the comparison clearly shows that several assumptions, for instance the historical period that is used, lead to different outcomes. These different outcomes have a direct and immediate consequence for government and the pensioning/insurance sector regarding the allocation of current and future resources.

The purpose of this study is to review the current methods behind official mortality forecasts in Europe, compare outcomes and assumptions of different projection methods within one country and compare the outcome of different types of methods for this country using similar explicit assumptions, including the same historical period. As a case study we do so for the Netherlands. We look at two measures: the life expectancy at birth and, in light of pension reforms, the life expectancy at age 65.

### **Data and methods**

After a first review of the current methods for forecasting mortality by statistical offices in Europe and the different national and international forecasts/projections that exist for the Netherlands, we shall show to what extent different methods applied to Dutch data for the period 1970 to 2009 lead to different future values of the life expectancy at birth ( $e_0$ ) and at age 65 ( $e_{65}$ ) up to 2050.

The methods for mortality forecasting by statistical offices in Europe and for the Netherlands are found in (internet) publications up to 2011, by using as main search terms (mortality) forecasting and (population) projection. Information on the method is given for Austria, Belgium, Denmark, England, France, Italy, Ireland, Luxemburg, Netherlands, Norway, Poland, Portugal, Spain and Sweden. Mortality forecasts for the Netherlands are published by Statistics Netherlands, the Actuarial Society,

RIVM (National Institute for Public Health and the Environment), Eurostat and three scientific projects TOPALS, UPE and Janssen, van Wissen and Kunst (forthcoming).

The six different methods with fundamentally different approaches we applied to the Dutch data for 1970 to 2009, for both sexes separately, are:

- direct (linear) extrapolation;
- the Lee-Carter model (Lee and Carter, 1992);
- an extension of the Lee-Carter model which includes the mortality experiences of other countries (Li and Lee, 2005);
- an extension of the Lee-Carter model which includes a cohort dimension (Renshaw and Haberman, 2006);
- a model in which smoking-related and non-smoking-related mortality is projected separately (Janssen and Kunst 2010; Janssen, van Wissen and Kunst forthcoming);
- and the methodology used in the official forecast by Statistics Netherlands.

We do not explicitly apply a method within the expectation approach because of the large dependence on expert-opinion in target setting, e.g. every outcome can be set. See section 2.3 for a more detailed description of these methods.

The choice to include data over the period 1970 to 2009 depended on the data requirements behind the methodology by Statistics Netherlands.

Next to keeping the historical period fixed, we shall use the observed values for 2009 as the jump-off rates for all the different methods. If possible, further specifications and assumptions within each framework are kept the same as well.

Data on all-cause mortality and population numbers by sex, age (0, 1-4, 5-9, ..., 90-94, 95+), and year (1970-2009) were obtained through Statistics Netherlands. For the Li-Lee model, in addition, the same data were obtained for Denmark, England & Wales, Finland, France, Italy, Norway, Spain, Sweden, Switzerland and West Germany through the Human Mortality Database. Also, lung cancer mortality data and cause-specific mortality data were obtained from Statistics Netherlands, for the separate projection of smoking- and non-smoking-related mortality and the official forecast, respectively.

## **Results**

### *Different ways of forecasting in Europe*

Mortality projection methods and assumptions clearly differ both by country and within countries. Within the context of new mortality projection methodologies with a focus on extrapolation, statistical offices in Europe nowadays indeed mostly predict mortality through extrapolation methods (either direct or Lee-Carter), but also make use of target values, expert opinion and cause-specific mortality projections (see Table 1). The method and the included historical period seem to reflect the past mortality trends in the country, with extrapolation suiting countries with gradual increases in life expectancy, and other approaches or different assumptions for countries with non-linear trends, such as the Netherlands.

### *Different forecasts/projections for the Netherlands*

Approaches of national and international projects for the Netherlands also include extrapolation methods, but mainly methods taking into account the nonlinearity in the observations by including trends in other countries, by the separate projection of smoking- and non-smoking-related mortality, or by projections by cause of death (see Table 2). The nine different projections for the Netherlands resulted in a large range for the life expectancy at birth in 2050 of almost 5.5 years for both males and females, which can be due to the different methods and to the different explicit assumptions, including the historical period.

### *Results of applying different methods to Dutch mortality from 1970 to 2009*

When applying different methods to the same historical period, a different range of outcomes can be expected. Thus, we apply similar methods which are used in Europe and the Netherlands to Dutch mortality data over the period 1970 to 2009 and compare the outcomes.

To recap, the methods applied to the Dutch data range from ‘simple’ extrapolation models (direct extrapolation and Lee-Carter) to models which account for non-linearity in the data, by either the inclusion of cohort effects or the inclusion of trends in other populations in the Lee-Carter model, or by more explanatory approaches, i.e. the separate projection of smoking and non-smoking mortality and the projection by cause-of-death as used in the official Dutch forecasts. See Data and methods for more details.

Direct linear extrapolation results in a life expectancy at birth in 2050 of 86.5 years for females and 83.3 years for males (Figure 1, Table 3). This means an increase of 3.8 years for women and 4.5 years for men over the next forty years. The Lee-Carter method gives higher life expectancy values, i.e. 87.4 years for females and 83.8 years for males. The Li-Lee model leads to 87.7 years for females and to the highest values for males (85.0 years). The cohort model gives a life expectancy at birth of 87.8 years for females and 83.5 for males. The “smoking+non-smoking” model, in which smoking-related mortality and non-smoking-related mortality are separately projected, leads to the highest predicted values of 88.6 years for females and 84.2 years for males. Statistics Netherlands, which uses a cause-of-death decomposition, projects a life expectancy at birth of 86.6 years for females and 83.7 years for males in 2050.

The difference in the life expectancy at birth for 2050 between the models is thus 2.1 years for females and 1.8 years for males. The average increase of the life expectancy at birth between 2009 and 2050 is 4.8 years for females and 5.4 years for males. The direct extrapolation model results in a lower life expectancy for both men and women than the other models. The methods which account for the non-linearity give generally higher outcomes than the ‘simple’ extrapolation models.

The increase is almost a straight line for the extrapolation methods, whereas the cohort model, the “smoking+non-smoking” model and the method of Statistics Netherlands are less linear (Figure 1). The straight line of the extrapolation methods is a result of linear, but slightly declining, increases in the life expectancy at birth over the period 2009-2050. The yearly increases of the cohort model, the “smoking+non-smoking” model and the method of Statistics Netherlands differ from year to year. They are not only different from the extrapolation methods, but also from each other (see Table 3 and compare the observation in 2009 and the outcomes in 2030 and 2050). For instance, the cohort model predicts for men a low increase in the life expectancy at birth over the period 2009-2030 compared to all other methods and a relatively high increase in the period 2030-2050. The “smoking+non-smoking” model predicts, for women, rising increases in the first half and declining increases in the second half of the period which results in the same increase over the period 2009-2030 as the period 2030-2050. The method of Statistics Netherlands predicts higher yearly increases in the first half of the period than the other methods and constant increases in the second half for both males and females.

Three of the six methods, i.e. the Lee-Carter model, the cohort model and the “smoking+non-smoking” model, forecast an increasing gap in the life expectancy at birth between the sexes.

When comparing the forecasted values of remaining life expectancy at age 65 in 2050 for the different forecasting methods, differences amount to 1.4 years for females, and 1.9 years among males (Figure 2, Table 3). Just as with the life expectancy at birth, the “smoking+non-smoking” model results in the highest remaining life expectancy at 65 for females (25.0 years) and the Li-Lee model results in the highest value for males (22.0 years). The direct extrapolation model results in the lowest value for both females and males (23.7 and 20.2 years, respectively).

The largest difference between the outcomes at birth and at age 65 is seen in the cohort model for females where the increase in life expectancy at age 65 is higher in comparison with the other methods. For males, the outcomes at age 65 drift apart more than the outcomes at birth.

## **Conclusion**

In this paper, we observed various different existing projection methodologies behind official mortality forecasts in Europe and in the Netherlands. The methods and the chosen historical period signify to be specific for the observed past trend in a country, with for the Netherlands – among others – methods that include the non-linearity observed in the past trends.

The different projection methods lead to different results, which can have enormous implications for the insurance and pension industries. However differences in outcomes are smaller when the same explicit assumptions are being used, such as the same historical data and the use of observed jump-off rates. The remaining sensitivity is small compared to other forms of uncertainty and to the increase in life expectancy over the long time horizon.

Remaining differences in outcome, for the Netherlands, especially show the differences between methods that include the observed non-linearity and simple extrapolation methods which do not. The first methods reveal a less linear trend in the future life expectancy and generally higher outcomes, which again can be linked to the past trends. However, because the outcomes can differ by the used historical period and other explicit assumptions, the resulting range might also be affected.

Because the choice of explicit assumptions add more to the differences than the choice of the forecasting approach, the choice for the projection method should not only be based on different approaches, but more importantly on the explicit assumptions. Especially within a country tuning in on these explicit assumptions, such as the historical period being used, by the different institutions performing mortality projections might be the best way forward, but may be difficult to realize.

As regards to the choice for the projection method some important considerations remain as well. Including additional information into the method automatically leads to more assumptions and thus more subjectivity, but, in the case of non-linear past trends is likely to result in more logical and more robust results. If past trends in mortality have been largely linear, though, it is better to use the simple extrapolation methods, especially because the outcomes of extrapolation methods using the same explicit assumptions do not differ much. All in all, a right balance between added information and added subjectivity needs to be aspired.

## Tables and Figures

*Table 1 Methods and assumptions behind the mortality forecasting methods of the statistical offices in Europe*

Country	Type of method	Assumptions	Historical period	Forecasted period
Austria	Direct extrapolation	Extension: using more recent data for the short term trend and convergence to a plausible function of age and sex for the long term (Ediev 2008)	1970-2008	2010-2050
Belgium	Direct extrapolation	Extension: Old-age adjustment; Extension: 10-year period for re-estimation after smoothing the age specific parameter	1970-2007	1990-2060
Denmark	Lee-Carter	Variant: adjust $k(t)$ to fit the observed life expectancy (Lee and Miller 2001); Extension: short term correction to account for the error between estimated jump-off rate and observation (Bell 1997); Extension: smoothing mechanism (Jong and Tickle 2005),	1990-2009	2010-2100
England	Target value, Expert opinion	Annual rate of improvement converges to 1.0 per cent in 2033 and remain constant thereafter; Variant: partly cohort approach for convergence	1900-2008	2008-2083
France	Direct extrapolation, Expert opinion	Age 3-13 no improvement after 2040; Extension: Old-age adjustment	1988-2002	2007-2060
Italy	Lee-Carter	Variant: an autoregressive time-series model with a deterministic time trend	Unknown	2001-2051
Ireland	Target value, Expert opinion	Long term rate of 1.5 per cent per annum from 2031 onwards; Extension: linear interpolation between mortality declines in 2005 en 2031	1926-2005	2011-2041
Luxembourg	Target value	Assumptions from Eurostat (convergence in 2100 for all EU countries; BMS method for total group)	1962-2005	2005-2055
Netherlands	Cause of death, Direct extrapolation, Lee-Carter, Expert opinion	Different assumptions per cause of death, depending on historical trend	1970-2009	2010-2060
Norway	Lee-Carter	Extension: a quadratic age effect	1900-2008	2010-2060
Poland	Target value	'Catch up' with developed countries 21-22 years later	1950-2005	2008-2035
Portugal	Lee-Carter, Expert opinion	Variant: Poisson log-bilinear model (Brouhns et al. 2002, Bravo 2007)	1980-2007	2008-2060
Spain	Direct extrapolation	5-year moving average; Extension: 3-year period for re-estimation after smoothing the age specific parameter	1991-2007	2009-2049
Sweden	Lee-Carter	Variant: no correction for time component	1990-2002	2003-2050

Hanika (2010); Bureau fédéral du Plan (2009); Frank Hansen and Stephensen (2010); Office for National Statistics (2009); Blanpain and Chardon (2006); Salvini, Santini and Vignoli. (2006); Statistiska centralbyrån (2005); Central Statistics Office (2008); STATEC (2005); Van Duin et al. (2011), De Jong and van der Meulen (2005); Keilman and Pham (2005); Główny Urząd Statystyczny (2009); Instituto Nacional de Estadística (2009); Instituto Nacional de Estadística (2009); Brunborg and Texmon (2010).

*Table 2 Methods, assumptions and outcomes (e0 in 2050) of different national and international mortality forecasts/projections for the Netherlands*

Forecast / projection by	Method	Assumptions	Historical period	e0 2050 NL		Difference
				Females	Males	
Eurostat	Lee-Carter, Target approach	Convergence mortality rates in 2100 for all EU countries; Variant: BMS method for total group	1962-2005	88.0	84.0	4.0
TOPALS 1	Target approach	Assumptions on future relative risks (the ratio between the transition rates of the country and those according to the standard age schedule) related to a standard age schedule in the base year	2003	83.6	80.2	3.4
TOPALS 2	Direct extrapolation	Variant: projection of the national age pattern in the base year using a random walk model with drift and a linear spline function	1985-2003	82.7	81.2	1.5
TOPALS 3	Coherent forecasting	Projection of a standard age schedule using a random walk model with drift and a linear spline function combined with assumptions on the relative risks relative to the projected standard age schedule	1985-2003	86.2	83.2	3.0
UPE	Target approach	The same rate of decline for all countries in 2030 (the eventual rate of decline was empirically estimated using eleven countries during a 30-year period). Extension: the change to the eventual rate is linear	1967/1971-1997/2001	86.4	82.5	3.9
Actuarial Society	Direct extrapolation	Two-year moving average; Extension: old-age adjustment; Extension: correction females	1988-2008	87.3	85.5	1.8
Statistics Netherlands	Direct extrapolation, Lee-Carter, Expert opinion, Cause of death	Different assumptions per cause of death, depending on the historical trend	1970-2009	86.6	83.7	2.9
RIVM	Explanatory approach, Coherent forecasting	Separate projection of smoking- and non-smoking-related mortality; Including mortality experiences of 10 other European countries	1970-2006	88.1	83.8	4.3
Janssen, van Wissen and Kunst (forthcoming)	Explanatory approach, Coherent forecasting	Separate projection of smoking- and non-smoking-related mortality; Including mortality experiences of 10 other European countries	1970-2006	87.4	83.6	3.8

Eurostat (2007); De Beer et al. (2007); Alders et al. (2007); Actuarieel Genootschap & Actuarieel Instituut (2010); Van Duin et al. (2011), De Jong and van der Meulen (2005); Janssen and Kunst (2010); Janssen, van Wissen and Kunst (forthcoming).

Table 3 Life expectancy at birth and at age 65: observed (2009) and projected values (2030, 2050) for different mortality forecasting methods, The Netherlands, by sex

	Females		Males		Sex difference	
	e0	e65	e0	e65	e0	e65
Observed values 2009	82.65	20.77	78.53	17.41	4.12	3.36
Projected values 2030						
Direct Extrapolation	84.73	81.23	22.32	18.92	3.50	3.40
Lee-Carter	85.25	81.59	22.73	19.16	3.67	3.57
Li-Lee	85.39	82.05	22.74	19.81	3.34	2.92
Cohort model	85.47	80.78	22.99	18.44	4.69	4.55
“smoking+non-smoking” model	85.61	81.77	22.62	19.69	3.84	2.92
Statistics Netherlands	84.90	81.73	22.40	19.72	3.17	2.68
Projected values 2050						
Direct Extrapolation	86.49	23.68	83.26	20.16	3.23	3.52
Lee-Carter	87.39	24.41	83.82	20.55	4.57	3.86
Li-Lee	87.72	24.50	85.02	22.03	2.70	2.47
Cohort model	87.80	24.87	83.45	20.27	4.35	4.60
“smoking+non-smoking” model	88.59	24.96	84.18	21.48	5.28	4.41
Statistics Netherlands	86.57	23.59	83.65	21.07	2.92	2.52

Figure 1 Life expectancy at birth; observed (1970-2009) and projected (2010-2050) values for different mortality forecasting methods, The Netherlands, by sex

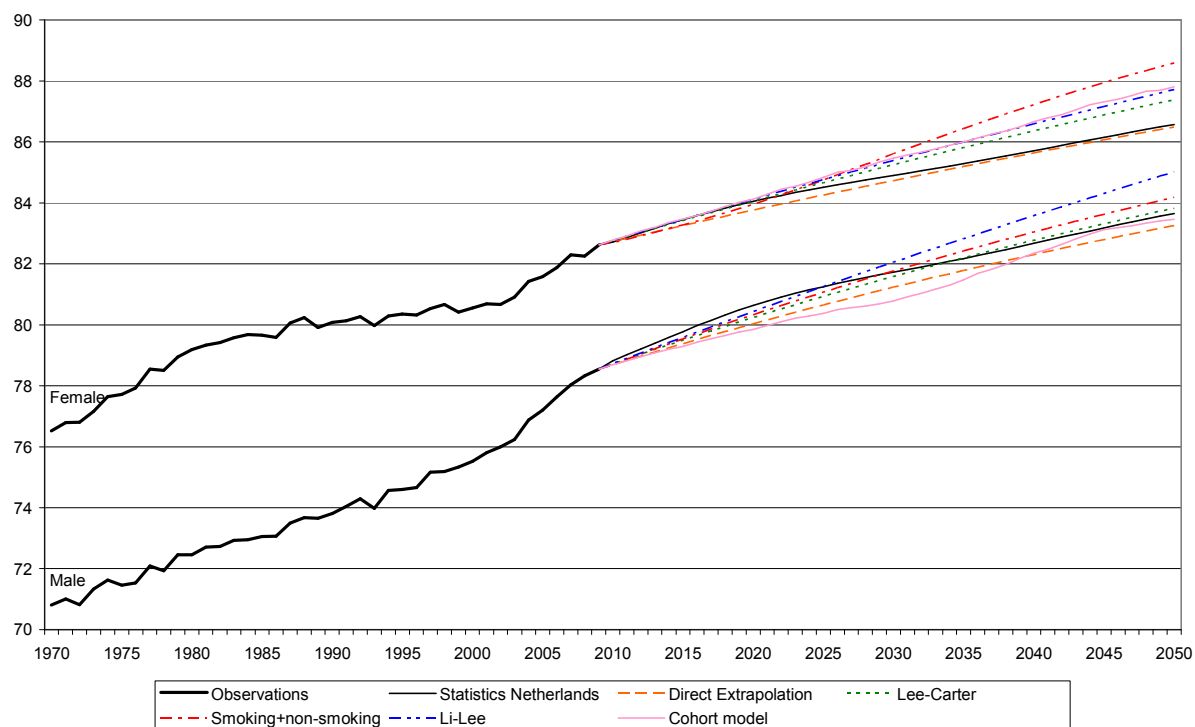
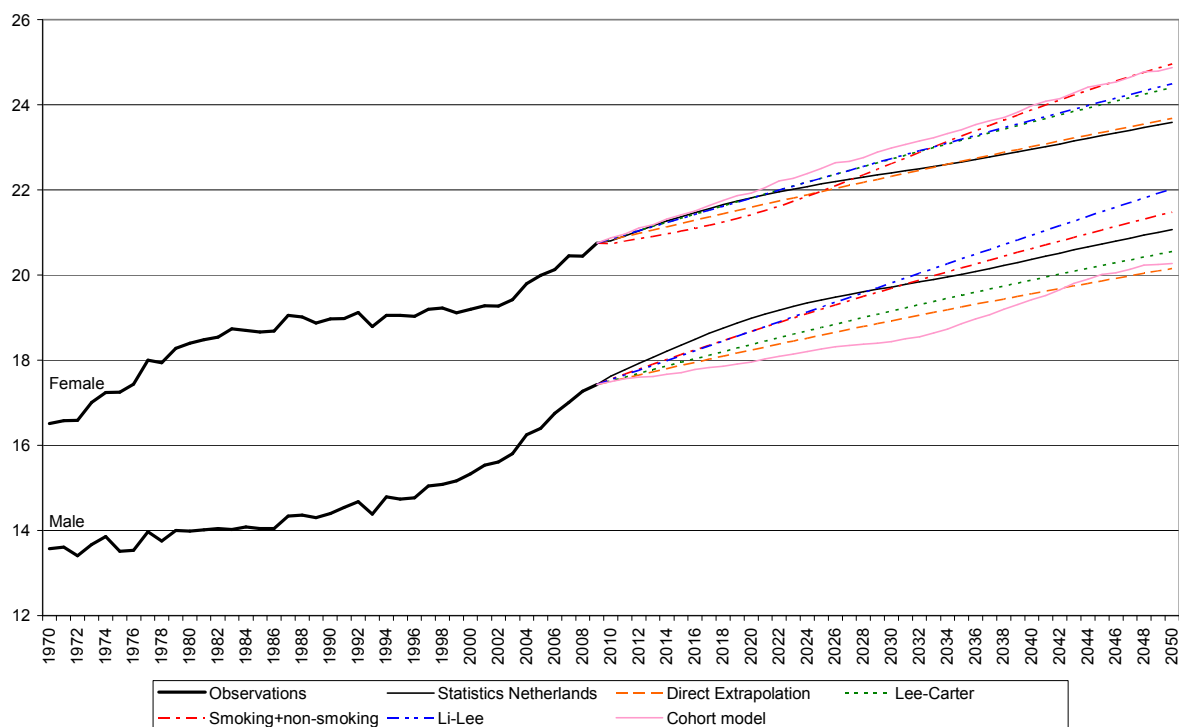


Figure 2 Life expectancy at age 65; observed (1970-2009) and projected (2010-2050) values for different mortality forecasting methods, The Netherlands, by sex



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